Pricing LYONs under stochastic interest rates

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Abstract: In this study, one of the simplifying assumptions of the McConnell and Schwartz (1986) LYON pricing model is relaxed. We present a valuation model that incorporates stochastic interest rates. LYON prices are computed with the modified explicit finite differences method of Hull and White (1990) and with the Least-Squares Monte Carlo technique. For the Waste Management issue, we find that the value of the LYON is very sensitive to the market price of interest rate risk and somehow sensitive to the correlation coefficient between interest rates and stock returns.

Key words: convertible bonds, option pricing, stochastic interest rates.

JEL Classification: C63, E43, G13.

Resumen: En este trabajo, relajamos uno de los supuestos simplificativos del modelo de valoración de LYONs desarrollado por McConnell y Schwartz (1986). Presentamos un modelo que incorpora tipos de interés estocásticos. El precio del LYON lo obtenemos utilizando el método de diferencias finitas modificadas de Hull y White (1990) y la técnica Least-Squares Monte Carlo. Para la emisión realizada por Waste Management, encontramos que el valor del LYON es muy sensible al precio de mercado del riesgo de tipo de interés, y algo sensible al coeficiente de correlación entre los tipos de interés y las rentabilidades de la acción del emisor.

Palabras clave: Bonos convertibles, valoración de opciones, tipos de interés estocásticos.

Clasificación JEL: C63, E43, G13.

1. INTRODUCTION

Liquid yield option notes, LYONs, are zero coupon convertible bonds that can be called by the issuer or redeemed by the investor at different prices through time. It is a proprietary product of Merrill Lynch, that created it in 1986. The complexity of the bond makes it more difficult to value and to sell, and the underwriter typically compensates this charging a higher underwriting fee (see Becker and Long, 1997).

* I would like to thank an anonymous referee for his helpful comments. All errors are my own.

This security has been priced, in a simple and practical way, by McConnell and Schwartz (1986) (MS hereafter). They assume that the value of a LYON depends on the issuer's stock price and that the interest rate is constant. These assumptions allow them to develop a model that captures most of the features of this security. However, as they point out, these assumptions may have an important impact on the pricing of LYONs.

The first assumption precludes the possibility of bankruptcy and overstates the value of the LYON, while the second assumption has a mixed effect. Uncertain interest rates would increase the value of the put and call features for the investor and the issuer, respectively. Thus, by assuming constant interest rates, the model understates (due to the put feature) and overstates (due to the call feature) the value of the LYON. Consequently, the net effect of stochastic interest rates will depend on the characteristics of the LYON.

MS also assume that the conversion and redemption strategies followed by investors and the call strategy followed by the issuer are "optimal", i.e. investors follow conversion and redemption strategies that maximize the value of the LYON, and the issuer follows a call policy that minimizes the value of the LYON. However, the empirical evidence[1] seems to indicate that the call policies of most firms are not optimal.

In this paper, we develop a more sophisticated model for pricing LYONs, incorporating stochastic interest rates. We find that the value of the LYON can increase substantially with the market price of interest rate risk and the correlation between interest rates and stock returns.

This article is organized as follows. Next section presents the valuation model. In Section 3, we apply the model to the pricing of a particular LYON issue. Finally, we conclude with Section 4.

2. THE MODEL

For completeness, we first present the MS model. They assume that the value of the LYON, L, depends upon the issuer's stock price, S, which follows a geometric Brownian motion process. That is,

$$dS(t) = \left(S\mu_s - D(S,t)\right)dt + S\sigma_s dZ_S \qquad [1]$$

where μ_s is the drift of the stock price process, D(S, t) is the total amount of dividends paid at time t, and σ_s is the instantaneous volatility of stock returns. MS use standard arbitrage arguments and show that the value of the LYON must satisfy the following partial differential equation (PDE)

$$\frac{1}{2}\sigma_s^2 S^2 L_{ss} + \left(Sr - D(S,t)\right)L_s + L_t - rL = 0, \qquad [2]$$

where r is the instantaneous risk-free interest rate (assumed to be constant in the model) and the subscripts of L represent partial derivatives. The boundary conditions for this equation are given by the different features of the security:

1 See Ingersoll (1977), for example.

- Maturity condition: L(S, T) = max(XS, F), where X is the conversion ratio (number of shares of the issuer's common stock into which the LYON can be converted), and F is the face value of the LYON.

- Conversion condition: L(S, t) ≥ XS. This is true at any time, t, prior to or equal to maturity, T.

- Put (or redemption) condition: $L(S, t_p) > P(t_p)$, where t_p are the times at which the LYON can be redeemed, and $P(t_p)$ are the redemption prices.

- Call condition: L(S, t) ≤ max(C(t), XS), where C(t) is the call price of the LYON at time t ≤ T.

Because of the complexity of these boundary conditions, no closed-form solution for the value of the LYON is known. MS use finite differences[2] to solve equation (2) and compute LYON prices.

In this paper, we extend the MS model and assume that interest rates are stochastic. To avoid complexity, we suppose that the term structure of interest rates is given by the instantaneous interest rate, whose dynamics can be described by the following stochastic differential equation (SDE)

$$dr = \alpha(\mu_r - r)dt + r\sigma_r dZ_r. \qquad [3]$$

Here, the interest rate converges to its long term mean, μr, at the velocity rate of α, and the conditional volatility of changes in the interest rate is proportional, with coefficient σ_r, to r.

Expressions (1) and (3) are related trough ρ, the instantaneous correlation coefficient between changes in interest rates and changes in stock returns; that is $dZ_s dZ_r = \rho dt$.

Thus we use the Brennan and Schwartz (1980) one-factor interest rate model, which belongs to the family of models given by $dr = \alpha(\mu_r - r)dt + r^{\gamma}\sigma_r dZ_r$ Our choice of γ = 1 can be justified by the empirical evidence of Chan et al. (1992), Uhrig and Walter (1996), and Navas (1999), whose estimates of g are 0.77, 1.50, and 1.77 for the German, U.S., and Spanish market, respectively.

We now assume that the value of the LYON depends upon the issuer's stock price and the instantaneous interest rate, which follow the SDE (1) and (3), respectively.

Using no arbitrage arguments, it is easy to derive the following PDE for the value of the LYON

$$\begin{aligned} &\frac{1}{2}\sigma_s^2 S^2 L_{ss} + \frac{1}{2}\sigma_r^2 r^2 L_{rr} + rS\rho\sigma_r\sigma_s L_{rs} + (Sr - D(S,t))L_s \\ &+ (\alpha(\mu_r - r) - \lambda\sigma_r r)L_r + L_t - rL = 0, \end{aligned} \qquad [4]$$

where λ represents the market price of interest rate risk.

To compute LYON prices, we solve equation (4) numerically, subject to the corresponding boundary conditions.

2 They do not specify whether they use explicit or implicit finite differences.

Although we do not intend to perform a formal study of alternative numerical techniques for valuing LYONs, we price them using two methods.

The first one is finite differences. Geske and Shastri (1985) compare finite difference methods with binomial trees for pricing options. They find that, although the binomial model is more intuitive and easy to implement, finite differences are more efficient. This is particularly true in two instances: a) when valuing American options and b) when the underlying asset pays a constant dividend yield. Note that both situations take place for LYONs. Geske and Shastri also argue that explicit finite differences are more efficient than implicit ones, since they do not require to solve systems of simultaneous equations. Moreover the explicit method is simpler because it does not require the inversion of matrices. As Hull and White (1990) (HW hereafter) point out, the only disadvantage of the explicit method is that the numerical solution does not necessarily converge to the solution of the PDE as the time step used tends to zero. Consequently, they develop a modified version that uses a transformation of variables and a new branching process that ensures convergence. In this paper, we use the modified model of Hull and White, extended to the case of two state variables: the stock price and the instantaneous interest rate.

The second numerical technique that we implement is Least-Squares Monte Carlo (LSM). This method, developed by Longstaff and Schwartz (2001), allows the pricing of American options by simulation, and it is especially useful when the value of the option depends on multiple factors. Since LYONs have American and Bermudan features and given that, in our proposed model, there are two sources of uncertainty, LSM seems a natural candidate among existing numerical techniques to obtain theoretical prices.

3. AN APPLICATION

We study one of the first LYON issues: the Waste Management issue on April 12, 1985. For a detailed summary of the characteristics of this particular product, see Table 1.

We first use the HW method to solve the PDE (2) for the case of constant interest rates. We assume that dividend payments are given by D(S, t) = y S, where y is the dividend yield of the stock. Table (2) shows that the MS model solved with the explicit finite difference method of Hull and White (1990) produces LYON prices (column 5) very similar to those reported by MS (column 4). The model overprices the Waste Management LYON by about $5 or 2%. The last column of the table reports LYON prices for the MS model using Least-Squares Monte Carlo. We simulate 100,000 (50,000 plus 50,000 antithetic) paths with 1,600 time steps before maturity. We use a constant and the first four Legendre polynomials evaluated at the stock price as basis functions on the current stock price[3]. For standard put options, Longstaff and Schwartz (2001) and Moreno and Navas (2003) show that, in general, no more than three polynomial term are needed to obtain accurate prices. The table shows that LYON values computed with LSM are lower than those obtained by finite differences. Interestingly, they are closer to market prices, although the overpricing does not disappear.

3 We have also used powers of the stock price instead of Legendre polynomials as basis functions, obtaining similar results.

Table 1

Put and call exercise prices and dates for the Waste Management, Inc. Lyon issue on April 12, 1985.

Date	Put price	Call price
12/04/1985	--	272.50
06/30/1986	--	297.83
06/30/1987	--	321.13
06/30/1988	301.87	346.77
06/30/1989	333.51	374.99
06/30/1990	375.58	406.00
06/30/1991	431.08	440.08
06/30/1992	470.75	477.50
06/30/1993	514.07	518.57
06/30/1994	561.38	563.63
06/30/1995	613.04	613.04
06/30/1996	669.45	669.45
06/30/1997	731.06	731.04
06/30/1998	798.34	798.34
06/30/1999	871.80	871.80
06/30/2000	952.03	952.03
01/21/2001	--	1000.00

The face value of each LYON is $1000, the maturity is January 21, 2001, the conversion rate is 4.36 shares of stock per bond. Waste Management may not call the bond prior to June 30, 1987, unless the price of their common stock rises above $86.01 (the stock price on April 12, 1985 was $52.25). After that date, Waste Management can call the bond at any time before than or at maturity. If the lyon is called between the dates shown in the table, the call price is adjusted (at a rate of 9% per year compounded semiannually) to reflect the interest accrued since the immediately preceding call date. The investor can elect to put the bond to Waste Management only on the dates given in the table.

Figures 1 through 3 present a sensitivity analysis of theoretical LYON values (using finite differences) to changes in the issuer's stock price, stock price volatility, dividend yield, and interest rate. We observe that the value of the LYON is not very sensitive to changes in the dividend yield, somehow sensitive to changes in the stock price volatility, and very sensitive to changes in the interest rate (when the stock price is $46, as the interest rate decreases from 0.1121 to 0.0721, the LYON price increases by 15.1%).

We next apply the stochastic interest rate (SIR hereafter) model to the Waste Management LYON. For the stock price process we use the parameters of the MS model, that is $\sigma_S = 0.30$ and

Table 2

Waste Management LYON prices for the constant interest rate case.

Date	Stock price	LYON market price	MS	MS(HW)	MS(LSM)
04/12/1985	52.250	258.75	262.7	262.9	260.1
04/15/1985	53.000	258.75	264.6	264.8	262.0
04/16/1985	52.625	257.50	263.7	263.8	261.0
04/17/1985	52.000	--	262.1	262.2	259.4
04/18/1985	52.365	257.50	263.0	263.2	260.4
04/19/1985	52.750	257.50	264.0	264.1	261.3
04/22/1985	52.500	257.50	263.3	263.4	260.7
04/23/1985	53.250	260.00	265.3	265.3	262.6
04/24/1985	54.250	265.00	267.9	267.9	265.3
04/25/1985	54.250	265.00	267.9	267.9	265.3
04/26/1985	54.000	265.00	267.2	267.4	264.6
04/29/1985	53.750	260.00	266.6	266.7	263.9
04/30/1985	52.125	260.00	262.4	262.6	259.7
05/01/1985	49.750	252.50	256.7	256.8	253.9
05/02/1985	50.500	250.00	258.4	258.6	255.5
05/03/1985	50.750	252.50	259.0	259.2	256.0
05/06/1985	50.500	252.50	258.4	258.6	255.5
05/07/1985	50.875	255.00	259.3	259.6	256.3
05/08/1985	50.750	253.75	259.0	259.2	256.0
05/09/1985	51.250	255.00	260.3	260.4	257.6
05/10/1985	53.125	260.00	265.0	265.1	262.3

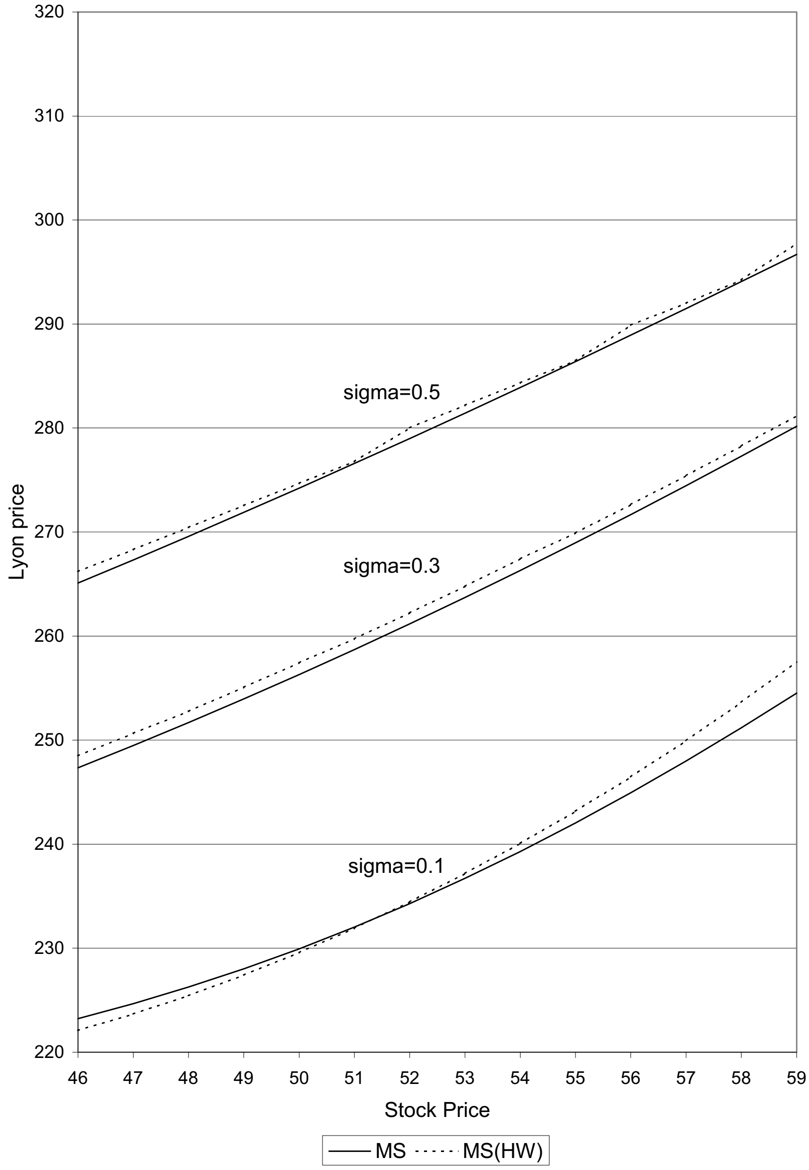
MS represents the LYON prices reported by McConnell and Schwartz (1986), while MS(HW) and MS (LSM) denote LYON prices when the McConnell and Schwartz (1986) model is solved using the explicit finite difference method of Hull and White (1990) and the Least-Squares Monte Carlo technique, respectively. The parameters of the model are dividend yield y = 0.016, stock price volatility σ_s = 0.30, and current interest rate r = 0.1121.

y = 0.016. For the interest rate process we take the parameters from Chan et al. (1992). They use one-month U.S. Treasury bills from June, 1964 to December, 1989 to estimate the parameters of the Brennan and Schwartz (1980) model. Consequently, we have that α = 0.3142, μ_r = 0.0770, and σ_r = 0.3442. To be consistent with MS, we take as current interest rate[4] r = 0.1121.

4 In our model, this rate should be the one-month Treasury bill yield on April 12, 1985. Using the Fama (1984) data set, this rate is 0.0771 on April 30, 1985 (in annualized form). However MS use the yield of intermediate term bonds of the same risk as the Waste Management bond.

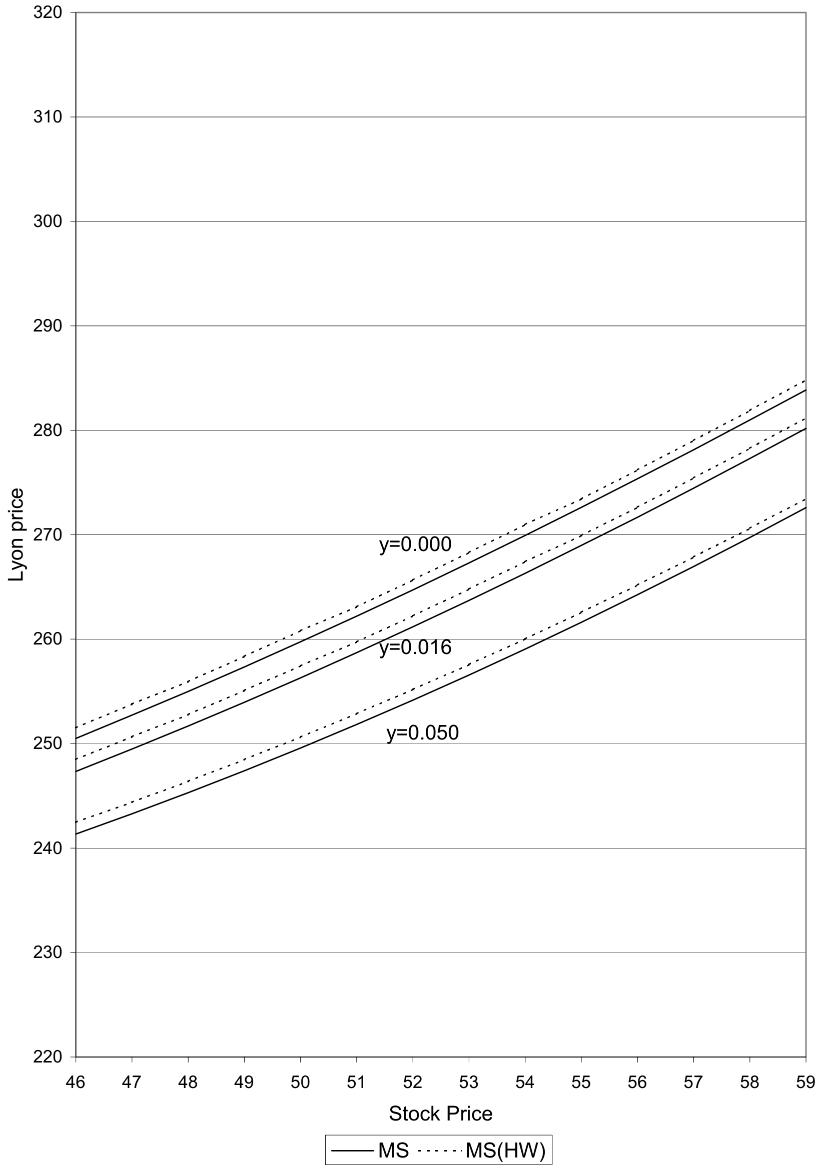
Figure 1

Sensitivity of LYON values in the constant interest rate case for changes in the stock price and the stock price volatility.



MS represents the LYON prices reported by McConnell and Schwartz (1986), while MS(HW) denotes LYON prices when the McConnell and Schwartz (1986) model is solved using the explicit finite difference method of Hull and White (1990). The other parameters of the model are dividend yield y = 0.016 and current interest rate r = 0.1121.

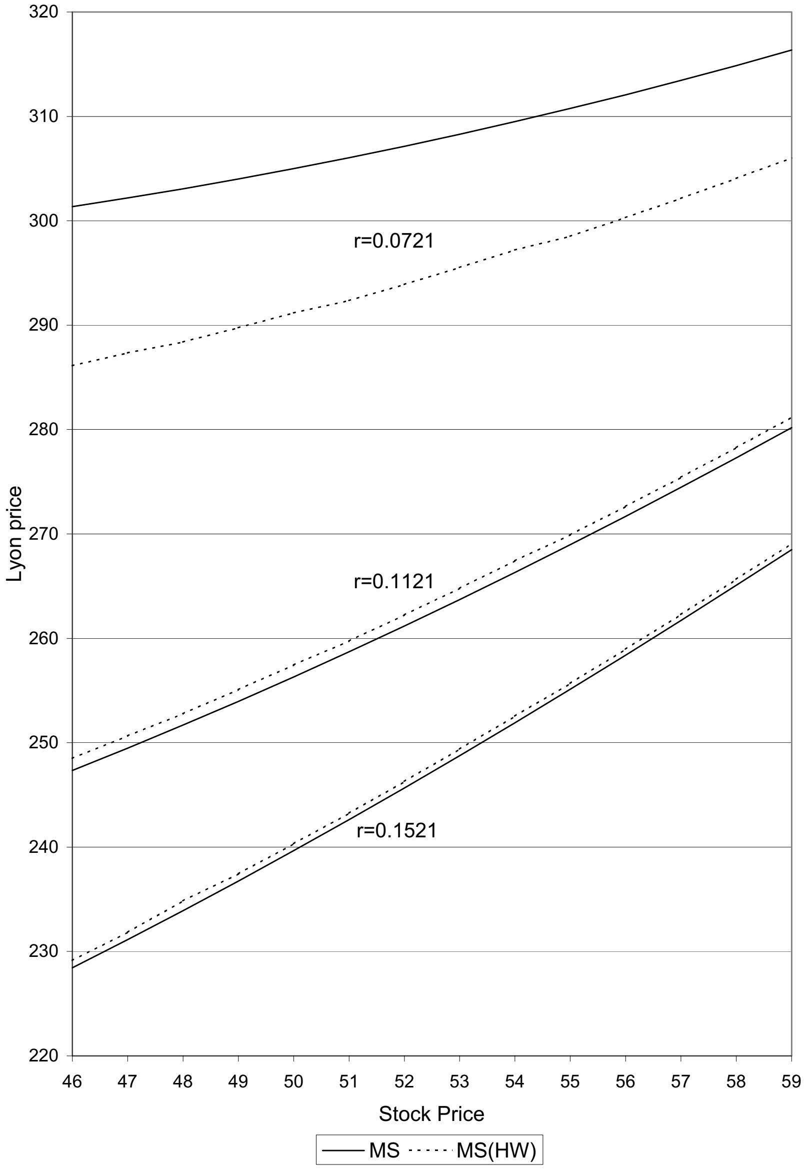
Figure 2
Sensitivity of LYON values in the constant interest rate case for changes in the stock price and the dividend yield.



MS represents the LYON prices reported by McConnell and Schwartz (1986), while MS(HW) denotes LYON prices when the McConnell and Schwartz (1986) model is solved using the explicit finite difference method of Hull and White (1990). The other parameters of the model are stock price volatility $\sigma_s = 0.30$ and current interest rate $r = 0.1121$.

Figure 3

Sensitivity of LYON values in the constant interest rate case for changes in the stock price and the interest rate.



MS represents the LYON prices reported by McConnell and Schwartz (1986), while MS(HW) denotes LYON prices when the McConnell and Schwartz (1986) model is solved using the explicit finite difference method of Hull and White (1990). The other parameters of the model are stock price volatility $\sigma_s = 0.30$ and dividend yield $y = 0.016$.

Table 3
Waste Management LYON prices with stochastic interest rates using the Hull-White method.

Stock price	Actual LYON price	MS(HW)	λ = -0.5, ρ = -0.2	λ = -0.5, ρ = 0.0	λ = -0.5, ρ = 0.2	λ = 0.0, ρ = -0.2	λ = 0.0, ρ = 0.0	λ = 0.0, ρ = 0.2
49	--	255.08	251.45	253.84	256.16	268.73	270.64	272.63
50	--	257.44	253.84	256.17	258.57	270.55	272.54	274.50
51	--	259.73	256.28	258.76	261.03	272.45	274.48	276.41
52	--	262.20	258.82	261.15	263.54	274.39	276.33	278.42
53	258.75	264.82	261.39	263.82	266.13	276.41	278.57	280.42
54	265.00	267.41	264.07	266.44	268.77	278.53	280.45	282.55
55	--	269.91	266.79	269.08	271.43	280.70	282.91	284.70

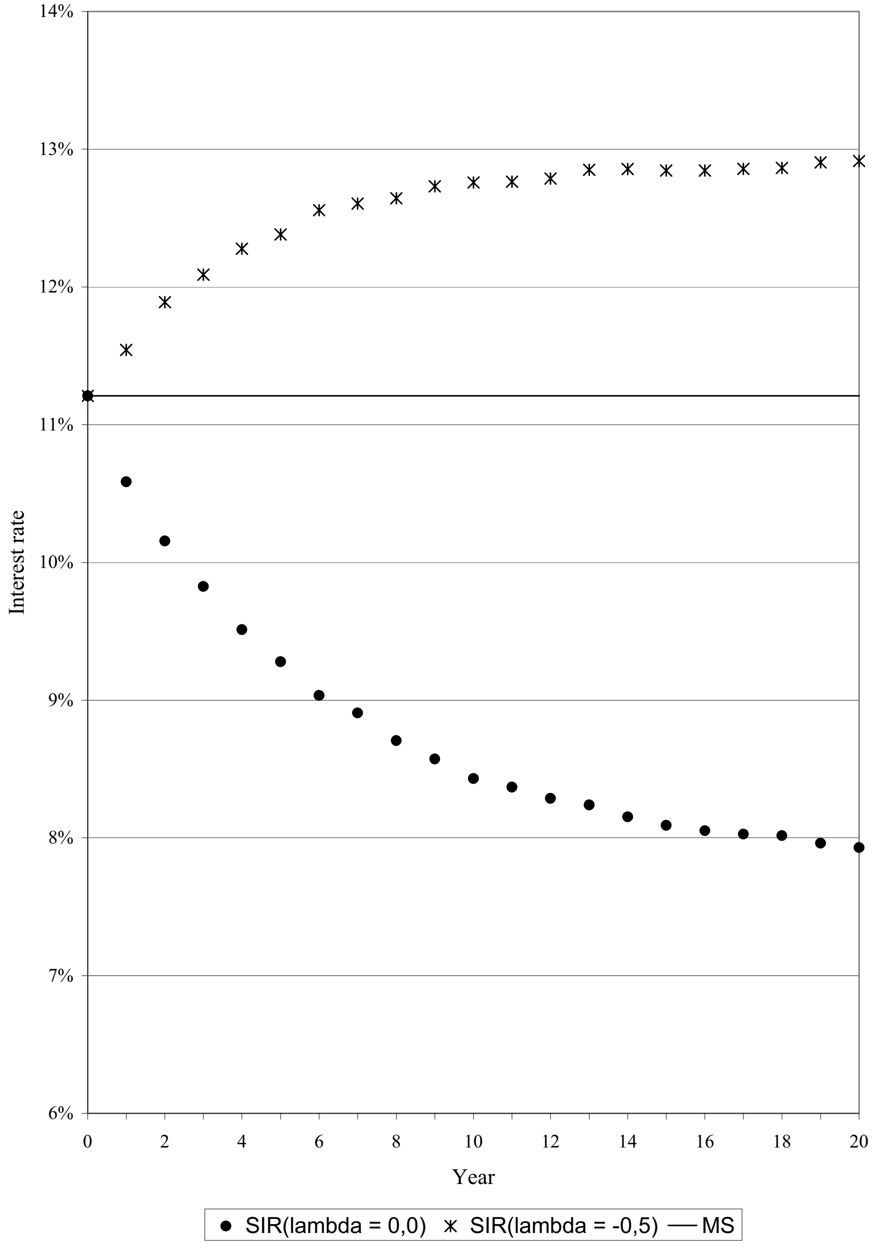
MS(HW) represents the McConnell and Schwartz (1986) constant interest rate model with the parameters reported in Table 2, implemented using the explicit finite difference method of Hull and White (1990). Columns 4 to 9 provide LYON prices for the stochastic interest rate model implemented with the finite differences method of Hull and White (1990), for different market prices of risk and correlation coefficients. The other parameters of the model are $\alpha = 0.3142$, $\mu_r = 0.077$, and $\sigma_r = 0.3442$.

Table 3 compares the stochastic and constant interest rate models using the finite differences method of HW. We observe that the SIR model (columns 4-9) produces a richer set of LYON prices than the MS model (column 3). For the former model we consider six cases, in which the market price of interest rate risk (λ) takes the values -0.5 and 0.0, while the correlation coefficient (ρ) takes the values[5] -0.2, 0.0, and 0.2. We see that LYON prices increase with λ and ρ. When λ = -0.5 and ρ = -0.2, the SIR model gives LYON prices that are lower than those of the MS model. If we increase ρ to 0.0, we obtain prices similar to those of the MS model. When ρ = 0.2, the SIR model gives prices higher than the MS model. When the market price of risk is zero, the SIR model overprices the LYON (relative to the MS model) for the three correlation coefficients used. For example, when the stock price is $49 and ρ = 0.0, the SIR model price is 6.9% higher than the MS price.

These results can be partially explained by the effect of λ on the shape of the initial yield curve. Figure 4 shows the term structure on interest rates implied by the Brennan and Schwartz (1980) model for different values of λ. The curves have been obtained numerically, computing the yield to maturity of a set of discount bonds. When the pure expectations theory of the term

5 In practice, the correlation coefficient is small. For instance, Brennan and Schwartz (1980) find that ρ = -0.01 for a sample of Treasury Bills and the CRSP market index.

Figure 4
Implied term structure of interest rates.



MS represents the constant interest rate model of McConnell and Schwartz (1986) (r = 0.1121), while SIR refers to the stochastic interest rate model with parameters $\alpha = 0.3142$, $\mu_r = 0.077$, and $\sigma_r = 0.3442$.

Table 4

Waste Management LYON prices with stochastic interest rates using Least-Squares Monte Carlo.

Stock price	λ					
	-0.5			0.0		
	ρ			ρ		
	-0.2	0.0	0.2	-0.2	0.0	0.2
49	257.34	259.77	262.67	262.21	264.98	267.57
50	259.35	261.75	264.50	263.86	266.64	268.95
51	261.30	263.82	266.60	265.55	268.28	270.54
52	264.12	266.18	268.62	267.80	270.02	272.00
53	266.25	268.33	270.47	269.57	271.68	273.55
54	268.53	270.40	272.45	271.74	273.56	275.30
55	270.91	272.72	274.41	273.72	275.39	277.00

Columns 2 to 7 provide LYON prices for different market prices of risk and correlation coefficients. The other parameters of the model are $\alpha = 0.3142$, $\mu_r = 0.077$, and $\sigma_r = 0.3442$.

structure holds and $\lambda = 0$, the yield curve starts at 0.1121 and falls fairly quickly to 0.077. As a consequence, LYON prices are higher than those in the MS model, even for low correlation coefficients. When $\lambda = -0.5$, the yield curve starts at 0.1121 and rises slowly (the 20-year interest rate is under 0.13). Thus LYON prices are lower than in the MS model, except for high correlation coefficients.

Finally, Table 4 shows LYON prices when the SIR model is implemented with Least-Squares Monte Carlo. We also simulate 100,000 paths with 1,600 time steps. We use the same basis functions as before, that is, a constant and the first four Legendre polynomials evaluated at the Waste Management stock price. As expected, LYON prices increase with λ and ρ, but now they are less sensitive to changes in λ. We see that, for the cases studied (λ and ρ ranging from -0.5 to 0.0 and from -0.2 to 0.2, respectively), LYON prices overstate those obtained with the MS model using the HW method.

The results of the LSM technique must be interpreted with care, since for complex securities the robustness of the algorithm to the choice of basis functions does not seem to be guaranteed (see Moreno and Navas, 2003). Note that LYON prices depend not only on the current stock price, but also on the whole term structure of interest rates. Thus, the basis functions for the regressions in the LSM method should probably include some polynomials evaluated at the current stock price, some others evaluated at current bond prices of different maturities, and cross products of these polynomials, to account for correlation between stock and bond prices. Unfortunately, there is no unique way to do this.

4. FINAL REMARKS

An important issue when pricing a bond (convertible or otherwise) is to match the initial yield curve. Using a simple one-factor interest rate model, this curve can be partially approximated. The assumption of constant interest rates can significantly affect theoretical LYON prices. McConnell and Schwartz (1986) find that their model overprices the Waste Management LYON on its first month of life. We show that decreasing the market price of risk or the correlation coefficient between interest rates and stock returns, we can obtain LYON prices consistent with market data. Of course, the same results could possibly be obtained with a model that allows for the possibility of bankruptcy of the issuing firm.

REFERENCES

— Becker, M. W. and M. S. Long (1997): "An Explanation of Underwriting Spread Differentials on Complex Securities", *Financial Management*, 26, 2, 35-43.

— Brennan, M. J. and E. S. Echwartz (1980): "Analyzing Convertible Bonds", *Journal of Financial and Quantitative Analysis*, 15, 4, 907-929.

— Chan, K. C., G. A. Karolyi, F. A. Longstaff, and A. B. Sanders (1992): "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate", *The Journal of Finance*, 47, 3, 1209-1227.

— Fama, E. F. (1984): "Term Premiums in Bond Returns", *Journal of Financial Economics*, 13, 529-546.

— Geske, R. and K. Shastri (1985): "Valuation of Approximation: A Comparison of Alternative Approaches", *Journal of Financial and Quantitative Analysis*, 20, 45-72.

— Hull, J. and A. White (1990): "Valuing Derivative Securities using the Explicit Finite Difference Method", *Journal of Financial and Quantitative Analysis*, 25, 1, 87-100.

— Ingersoll, J. E. (1977): "An Examination of Corporate Call Policies on Convertible Securities", *Journal of Financial Economics*, 32, 2, 463-478.

— Longstaff, F. A. and E. S. Schwartz (2001): "Valuing American Options by Simulation: A Simple Least-Squares Approach", *Review of Financial Studies*, 14, 1, 113-147.

— McConnell, J. J. and E. S. Schwartz (1986): "LYON Taming", *The Journal of Finance*, 41, 3, 561-576.

— Moreno, M. and J. F. Navas (2003): "On the Robustness of Least-Squares Monte Carlo (LSM) for Pricing American Derivatives", *Review of Derivatives Research*, 6, 107-128.

— Navas, J. F. (1999): "Consistent versus Non-Consistent Term Structure Models: Some Evidence from the Spanish market", *The Journal of Fixed Income*, 9, 3, 42-60.

— Uhrig, M. and U. Walter (1996): "A New Numerical Approach for Fitting the Initial Yield-Curve", *The Journal of Fixed Income*, 5, 4, 82-90.