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Portfolio Immunization using Independent Component Analysis

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ABSTRACT

Fixed-income portfolio managers usually use Principal Component Analysis (PCA) to find the lowest number of factors that explain the behavior of a set of term structure key rates at a specific confidence level. This technique reduces the size of the multidirectional immunization problem and improves the solution feasibility. Barber and Copper (1996) and Golup and Tilman (1997) are clear examples. However, when we use PCA factors in risk analysis we do not take into account the moments with order higher than two of the probability function of the yields, causing misspecification problems. In this paper we show how to apply the Independent Component Analysis (ICA), an alternative statistical technique to PCA, in fixed-income portfolio management in order to achieve this same objective but avoiding misspecification problems and improving the immunization results.

Keywords: Immunization; Principal Components Analysis; Independent Components Analysis; Multidirectional Durations.

JEL Classification: C39, E43, G11, G32.

Inmunización de carteras mediante análisis de componentes independientes

RESUMEN

Los gestores de carteras sobre renta fija usualmente emplean el análisis de componentes principales (PCA) para encontrar el mínimo número de factores que explican el comportamiento de un conjunto de tipos clave de estructura temporal a un nivel de confianza determinado. Esta técnica reduce el tamaño del problema de inmunización multidireccional y mejora la factibilidad de la solución. Balbás y Cooper (1996) y Golup y Tilman (1997) son ejemplos claros de esto. Sin embargo, cuando usamos los factores PCA en el análisis de riesgos no tomamos en consideración los momentos de orden superior a dos de la función de probabilidad de los rendimientos, originando problemas de nula especificación. En este trabajo mostramos cómo aplicar el análisis de componentes independientes (ICA), una técnica estadística alternativa a PCA, en la gestión de carteras de renta fija con el fin de lograr el mismo objetivo pero evitando problemas de nula especificación y mejorando los resultados de inmunización.

Palabras Clave: Inmunización; Análisis de Componentes Principales; Análisis de Componentes Independientes; Duraciones Multidireccionales .

Clasificación JEL: C39, E43, G11, G32.

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1. INTRODUCTION

Fixed-income portfolio managers use the financial immunization technique to protect the portfolio value against interest rate shifts, i.e. to minimize the impact of interest rate risk. Duration is the basic tool in this technique and approximates the exposure, under specific hypothesis, of cash flows present values to interest rate shifts. For a given portfolio, offsetting the duration of asset cash flows against the duration of the liability stream enables us to balance the effects of interest rate shifts, thus achieving immunization.

Alternative hypotheses on the behavior of the term structure of interest rates (TSIR) define alternative duration measures whose effectiveness depends on how close the underlying hypotheses are to the real evolution of the term structure, i.e. the inherent model risk. In order to distinguish these alternative duration measures they have different names, such as Macaulay duration, Fischer-Weil duration, stochastic duration, vector duration, factor duration, etc.

In this context, Reitano [1992] and Ho [1992] simultaneously develop two multidimensional duration measures: directional duration and duration with respect to a "key rate". Both concepts include cash-flow sensitivity to changes of a specific set of interest rates that define the whole term structure. In this sense, the greater the number of directions – or key interest rates, the closer the real sensitivity of the present value of cash-flows to the movements of the whole TSIR.

Although the explanatory level of TSIR behavior increases when we use an ever-greater number of interest rates, the cost is an ever-greater loss of freedom degrees to solve the immunization problem and, therefore, we need an ever-greater number of traded bonds to immunize a portfolio. In order to consider this trade-off, portfolio managers usually apply Principal Component Analysis (PCA). Specifically, they use PCA to find the lowest number of factors that explain the behavior of key rates at a specific confidence level. PCA enables us to reduce the size of the immunization problem and increase the solution feasibility. Barber and Copper [1996] and Golup and Tilman [1997] are clear examples of how to apply this technique to manage fixed-income portfolios.

An additional advantage of using PCA is that the factors extracted are orthogonal and therefore independent in a Gaussian environment. However, interest rates, like most financial variables, show leptokurtic behavior and the Gaussian behavior hypothesis is rejected. In this context, using PCA factors – which do not take into account the moments higher than order two, implies assuming misspecification errors. The aim of this paper is to show that using a statistical technique complementary to PCA we improve the immunization results by avoiding the Gaussian hypothesis in interest rates behavior implicit in PCA. This technique is Independent Component Analysis (ICA), a technique that physicists usually use to solve signal processing problems.

Given a set of non-Gaussian variables, such as key interest rates, that describes the TSIR, ICA allows us to extract the independent latent factors from their historical behavior. ICA is, therefore, a suitable technique to apply in the design of immunization strategies and in other portfolio management problems that already apply PCA successfully. Despite its suitability, ICA is not as popular as PCA in portfolio management and we hope the results of this article move academic and practitioners' analysis forward through the introduction of this technique.

2. MULTIDIRECTIONAL IMMUNIZATION

Zero coupon interest rates shifts are not perfectly correlated. This fact is the foundation of multidirectional immunization. Different factors explain co-movements of interest rates that correspond to different maturities. Following the Barber-Cooper terminology, a linear combination (with weights u) of a number (K) of independent factors (h) explains the variations (x) of interest rates (r) for each maturity (s) of a yield curve, i.e.:

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$$x\left(s\right)=r\left(s\right)-r_{0}\left(s\right)=\sum_{k=1}^{K}u_{k}\left(s\right)h_{k} \tag{1}$$

In addition, if $S\left(0\right)$ is the present value of a number $N$of portfolio cash-flows C, and P_i is the actual value of \$1 in moment $\left(t_i\right)$, then:

$$\begin{array}{l}P_{i}\left(0\right)=\exp\left[-r_{0}\left(t_{i}\right)t_{i}\right]\\ S\left(0\right)=\sum_{i=1}^{N}C_{i}P_{i}\left(0\right)\end{array} \tag{2}$$

From expressions 1.1 and 1.2, we can express the portfolio surplus value as a function on each of the explanatory factors (k) of the TSIR as:

$$\forall k=1,...,K \qquad \Delta S_{k}=\left[\sum_{i=1}^{N}C_{i}P_{i}\left(0\right)u_{k}\left(t_{i}\right)t_{i}\right]\Delta h_{k} \tag{3}$$

In this framework, multifactorial immunization requires that:

$$\forall k=1,...,K \qquad \Delta S_{k}=0 \tag{4}$$

On the other hand, we define cash flow portfolio sensitivity with respect to each independent factor or direction as:

$$\forall k=1,...,K \qquad D_{k}^{S}=\frac{\sqrt{N}}{S\left(0\right)}\left[\sum_{i=1}^{N}C_{i}P_{i}\left(0\right)u_{k}\left(t_{i}\right)t_{i}\right] \tag{5}$$

Therefore, in terms of duration, an immunized portfolio is one in which the cash flows have null multidirectional durations and an initial surplus value of \$0. The impact of factor changes on the value of asset and liability cash flows are equal with opposite signs and the portfolio surplus remains invariant to interest rates movements.

3. INDEPENDENT COMPONENT ANALYSIS

The use of ICA in the financial literature is relatively recent (Bellini and Salinelli [2003], Back and Weigend [1997]). For this reason, we introduce the main characteristics of ICA in this section. The ICA objective is to determine non-Gaussian latent factors and their weights on the data, using just two elements: historical data and factors independence. In contrast to PCA, the non-normal nature of latent components is intrinsic to ICA. As a matter of fact, ICA only relies on the crucial assumption that the underlying factors are statistically independent. This hypothesis is much stronger than uncorrelation when the factor is non-Gaussian (Ikeda [2000, a]).

The general ICA formulation is: if $z_{i,t}$is an observed value of asset $i=1,...,N$in time $t\in[1,\ T]$, we can express it as a linear mix of independent blinds or factors(f):

$$z_{i,t}=\sum_{j=1}^{N}a_{i,j}f_{j,t} \tag{6}$$

Where $a_{i,j}$ is the weight of the jth-factor on the ith-asset.

On the other hand, $y_{j,t}$ is the recuperated value of the jth-factor in the instant $t\in[1,\ T]$ by de-mixing matrix:

$$y_{j,t}=\sum_{i=1}^{N}w_{j,i}z_{i,t} \tag{7}$$

Where $w_{j,i}$ is the weight of the ith-asset on the jth-factor.

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The full process is composed of two stages: mixing and de-mixing. Separation is perfect when the recuperated factors are the same as the original factors. If separation is not perfect, we can express the factorial model as follows:

$$z_{i,t} = \sum_{j=1}^{N} a_{i,j} f_{i,t} + \varepsilon_{i,t} \tag{8}$$

Where the residual,$\varepsilon_{i,t}$, is Gaussian. The expression 1.8 is frequently used in the modeling of financial return fluctuations driven by latent factors.

Given that the ICA method uses normalized data, we need to transform the data before estimating the independent factors. Following Ikeda [2000, b], we first fix the maximum number of factors. We then avoid noise that could affect the independent factors estimation process (high-pass filtering) by taking log-difference prices. Thirdly, we eliminate the data mean (decentered). And finally, we eliminate the second moments (variance-covariance) or whitening stage.

There are three ways to estimate independent factors (see Hyvärinen, et al. [2001]). In this study we use the FastICA algorithm (Fast fixed-point algorithm) because it involves a minor computational cost and provides good results.

4. APPLYING ICA TO A HISTORY OF SHIFTS IN TERM STRUCTURE

For a better comparative illustration we will apply ICA to the case that Barber and Cooper [1996] propose in their paper. Following them, we use the McCulloch-Kwon [1993] TSIR database and select spot rates for thirty-nine maturities, ranging from one month to twenty years, for each month from August 1985 to February 1991. Table 1 shows the yields of February 1991, the three first eigenvectors obtained by Barber and Cooper [1996] from PCA and the respective weights obtained using ICA[1].

Then, we check ICA and PCA immunization performance. To do this we need to enlarge the database beyond February 1991 and therefore we use the U.S. Treasury monthly zero coupon curve[2] from March 1991 to February 1992 as a sample for testing.

In the Barber-Cooper case, the portfolio has three liability cash flows of \$100,000, \$200,000 and \$300,000 with maturities of 3, 4 and 5 years, respectively. The assets available to obtain an immunized portfolio have one, three, five and seven years to maturity, and the optimal asset cash flows are \$13,227.90; \$95,591.00; \$582,079.00; and \$-91,818.00. Through ICA we obtain the following asset cash flows that immunize the portfolio: \$-24,049.00; \$250,220.00; \$375,470.00; and \$-2,599.70; for one, three, five and seven years to maturity, respectively.

With respect to durations for the three directions in Barber-Cooper, we observe liability cash flows of 4.9248 (D1), -0.9822 (D2) and -1.6491 (D3), while asset cash-flow durations are 4.9251 (D1), -0.9829 (D2) and -1.6483 (D3). On the other hand, in the ICA case, the liability cash-flow durations are 3.2013 (D1), -0.3338 (D2) and -1.9495 (D3), and for asset cash flow durations are 3.2012 (D1), -0.3338 (D2) and -1.9496 (D3).

To verify whether the changes of present value of asset cash flows will balance the changes of present value of liability cash flows, i.e. to test immunization performance, we use twelve different planning periods with a length from one to twelve months.

Table 2 shows the present values of liability cash flows and asset cash flows for both alternative methods, PCA and ICA, and their respective surplus value. We can see that the range of values for PCA surplus (-429.19; 424.69) is larger than the range of values for ICA surplus (-320.19; 252.62), but the PCA surplus value is lower in March 1991, September 1991 and December 1991. However, to measure the degree of immunization of these portfolios we

[1] We report the ICA weights only for better option in this case, that is, without high-pass filtering and with third functions $\Psi(\cdot)$ of Appendix.

[2] http://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yield_historical_huge.shtml

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Maturity	Feb-91	PCA			ICA			Feb-90	PCA			ICA		
		U1	U2	U3	A1	A2	A3		U1	U2	U3	A1	A2	A3
0.083	5.68%	0.1065	0.3672	0.7775	0.1957	0.0945	0.3808	7.74%	0.1099	0.3858	0.7722	0.1888	0.0968	0.3871
0.167	6.00%	0.1047	0.2711	0.2773	0.1684	0.0127	0.2672	7.92%	0.1083	0.2766	0.2701	0.1626	0.0104	0.2689
0.25	6.18%	0.1129	0.2287	0.0606	0.1518	-0.0376	0.1881	7.98%	0.1158	0.2276	0.0537	0.1466	-0.0416	0.1875
0.333	6.23%	0.1238	0.2122	0.0128	0.1426	-0.0652	0.1355	7.96%	0.1256	0.2084	0.0067	0.1376	-0.0692	0.1342
0.417	6.21%	0.1329	0.1977	-0.0008	0.1367	-0.083	0.0962	7.95%	0.1337	0.1921	-0.0065	0.1319	-0.0865	0.0953
0.5	6.19%	0.1393	0.1792	-0.0241	0.1322	-0.0963	0.0647	7.95%	0.1398	0.1722	-0.0296	0.1279	-0.0992	0.0648
0.583	6.19%	0.144	0.161	-0.0532	0.1286	-0.1067	0.0389	7.97%	0.1444	0.1533	-0.0582	0.1248	-0.1092	0.0402
0.667	6.21%	0.1478	0.1463	-0.0806	0.1255	-0.1154	0.0176	7.98%	0.1482	0.1387	-0.0848	0.1222	-0.1174	0.0199
0.75	6.24%	0.1511	0.1372	-0.1008	0.1229	-0.1223	0.0003	7.99%	0.1514	0.1301	-0.1035	0.12	-0.1238	0.0033
0.833	6.29%	0.1541	0.1314	-0.1114	0.121	-0.1272	-0.0133	8.00%	0.1542	0.1253	-0.1127	0.1183	-0.1283	-0.0099
0.917	6.36%	0.1569	0.127	-0.1151	0.1196	-0.1299	-0.0238	8.00%	0.1568	0.1222	-0.1152	0.1171	-0.1308	-0.0204
1	6.43%	0.1594	0.1224	-0.114	0.1188	-0.1306	-0.0321	8.01%	0.159	0.1191	-0.1136	0.1164	-0.1315	-0.0289
1.083	6.51%	0.1615	0.117	-0.1112	0.1184	-0.1295	-0.0387	8.02%	0.161	0.1151	-0.111	0.1161	-0.1306	-0.0358
1.167	6.58%	0.1636	0.1103	-0.1076	0.1183	-0.127	-0.0441	8.05%	0.1629	0.1094	-0.108	0.1162	-0.1282	-0.0416
1.25	6.65%	0.1654	0.103	-0.1062	0.1183	-0.1236	-0.0489	8.08%	0.1647	0.1028	-0.1074	0.1163	-0.1249	-0.0467
1.333	6.71%	0.1671	0.0955	-0.1063	0.1182	-0.1197	-0.0533	8.11%	0.1663	0.0957	-0.1086	0.1164	-0.1211	-0.0512
1.417	6.77%	0.1687	0.0885	-0.1083	0.118	-0.1159	-0.0574	8.15%	0.168	0.0886	-0.1113	0.1164	-0.1172	-0.0554
1.5	6.81%	0.1702	0.0823	-0.1108	0.1177	-0.112	-0.061	8.18%	0.1695	0.0823	-0.1143	0.1162	-0.1131	-0.059
1.75	6.89%	0.1742	0.0684	-0.1194	0.1166	-0.1007	-0.0686	8.25%	0.1737	0.0678	-0.1225	0.1155	-0.1008	-0.0658
2	6.95%	0.1778	0.0589	-0.1198	0.1162	-0.088	-0.0701	8.27%	0.1776	0.0587	-0.1217	0.1155	-0.0869	-0.0663
2.5	7.07%	0.182	0.0354	-0.1088	0.1172	-0.0637	-0.0694	8.27%	0.1824	0.0369	-0.1105	0.1173	-0.0617	-0.0644
3	7.19%	0.1844	0.0105	-0.1053	0.1171	-0.0516	-0.0768	8.31%	0.1849	0.0118	-0.1068	0.118	-0.0495	-0.071
4	7.43%	0.1864	-0.0224	-0.0663	0.1176	-0.0253	-0.0749	8.36%	0.187	-0.0207	-0.0674	0.1188	-0.0252	-0.0712
5	7.62%	0.1825	-0.0561	-0.0486	0.1213	0.0042	-0.0707	8.39%	0.183	-0.0521	-0.0495	0.1229	0.0028	-0.0686
6	7.77%	0.1786	-0.0828	-0.0458	0.1216	0.022	-0.0791	8.41%	0.1791	-0.0804	-0.0444	0.1241	0.0196	-0.0775
7	7.86%	0.1755	-0.1061	-0.0208	0.119	0.0324	-0.0841	8.41%	0.1759	-0.1061	-0.0173	0.1217	0.0286	-0.084
8	7.94%	0.1725	-0.1296	0.0137	0.1152	0.0385	-0.088	8.41%	0.1727	-0.1317	0.0186	0.1176	0.0335	-0.0895
9	8.00%	0.1702	-0.1497	0.0455	0.1128	0.047	-0.0886	8.43%	0.1702	-0.1533	0.0507	0.1152	0.0414	-0.0908
10	8.07%	0.1685	-0.1658	0.0715	0.1123	0.0592	-0.0849	8.46%	0.1684	-0.17	0.0767	0.1147	0.0535	-0.0871
11	8.14%	0.1673	-0.1782	0.0926	0.1129	0.074	-0.0779	8.50%	0.1671	-0.1825	0.0979	0.1155	0.0685	-0.0797
12	8.22%	0.1663	-0.1876	0.1089	0.1142	0.0902	-0.0689	8.54%	0.166	-0.1916	0.1141	0.117	0.0851	-0.07
13	8.29%	0.1654	-0.194	0.121	0.1155	0.1066	-0.0588	8.59%	0.165	-0.1973	0.126	0.1186	0.1022	-0.059
14	8.35%	0.1644	-0.1975	0.1283	0.1164	0.122	-0.0486	8.63%	0.1641	-0.1998	0.1328	0.1198	0.1183	-0.0478
15	8.41%	0.1633	-0.1983	0.1302	0.1165	0.1352	-0.0395	8.66%	0.1628	-0.1992	0.1341	0.1201	0.1324	-0.0375
16	8.45%	0.1619	-0.1962	0.1261	0.1153	0.1451	-0.0323	8.68%	0.1612	-0.1955	0.1287	0.1191	0.1433	-0.0291
17	8.48%	0.1602	-0.1916	0.1158	0.1128	0.1511	-0.0274	8.70%	0.1592	-0.1893	0.1169	0.1167	0.1502	-0.0231
18	8.50%	0.1579	-0.1847	0.0997	0.109	0.1533	-0.0249	8.70%	0.1567	-0.1805	0.0987	0.113	0.1532	-0.0195
19	8.50%	0.1553	-0.1758	0.0795	0.1042	0.1518	-0.0247	8.69%	0.1537	-0.1697	0.0764	0.1082	0.1525	-0.0183
20	8.49%	0.1524	-0.1651	0.0568	0.0987	0.1468	-0.0266	8.66%	0.1504	-0.1571	0.0514	0.1026	0.1481	-0.0195

Table 1. ICA Yields, Eigenvectors and Weights for February 1991

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use the hedge ratio, defined in the International Accounting Standard number 39 (IAS 39) as: Changes in the present value of liability cash flows over changes in the present value of asset cash flows.

If the portfolio is perfectly immunized, this ratio will be equal to 100 %, but this ideal case is highly improbable due to the well-known drawbacks of the immunization technique and the model risk implicit in their specification. For this reason we take, as with IAS 39, the interval 80% to 125% of the values of the hedge ratio as an acceptable immunization degree. As Table 2 shows, only the PCA portfolio hedge ratio for a four months planning period is outside the IAS 39 range, reaching a value of 131.16%.

Date	Liability Value	Asset Value BARBER and COOPER	Asset Value ICA	Surplus ($) BARBER and COOPER	Surplus ($) ICA	Hedge (%) BARBER and COOPER	Hedge (%) ICA
Feb-91	434,101.83	434,101.65	434,101.59	-0.18	-0.25		
Mar-91	435,314.22	435,213.25	435,140.41	-100.97	-173.81	109.07%	116.71%
Apr-91	440,563.45	440,440.22	440,441.77	-123.23	-121.68	100.43%	99.02%
May-91	443,416.42	442,987.23	443,417.93	-429.19	1.51	112.01%	95.86%
Jun-91	442,781.82	442,503.40	442,821.95	-278.42	40.16	131.16%	106.48%
Jul-91	448,168.42	448,017.38	448,213.90	-151.04	45.48	97.69%	99.90%
Aug-91	459,570.88	459,467.47	459,476.61	-103.41	-94.27	99.58%	101.24%
Sep-91	469,742.09	469,833.65	469,586.98	91.56	-155.1	98.12%	100.60%
Oct-91	476,331.31	476,514.53	476,264.15	183.22	-67.16	98.63%	98.68%
Nov-91	484,190.28	484,614.98	484,110.14	424.69	-80.15	97.02%	100.17%
Dec-91	497,504.56	497,733.14	497,184.38	228.57	-320.19	101.50%	101.84%
Jan-92	491,736.41	492,107.69	491,989.03	371.29	252.62	102.54%	111.03%
Feb-92	492,744.07	492,938.52	492,819.97	194.45	75.9	121.28%	121.27%

Table 2. Immunization Results (Mar-91 to Feb-92)

Though the behavior of the TSIR in the testing sample and the restrictions on the asset and liability cash flows in the Barber and Cooper case could cause these results, we repeat the immunization test using an alternative problem and a testing sample with more significant movements of the TSIR that covers from October 2007 to September 2008. Now the estimation sample covers the period from January 1997 to September 2007[3]. Table 3 shows the TSIR interest rates for September 2007 and three first eigenvectors for PCA and ICA weights.

Now, the liabilities cash-flows to immunize are: $-400,000.00, $-100,000.00 and $-100,000.00; and their respective maturities: one, two and three years. The assets cash flow maturities remain the same in the Barber and Cooper case: one, three, five and seven years, and the values that immunize the portfolio are respectively:

1. For PCA: $442,510.00 (year 1), $155,930.00 (year 3), $14,149.00 (year 5) and $-12,985.00 (year 7).
2. For ICA: $435,770.00 (year 1), $178,370.00 (year 3), $-12,344.00 (year 5) and $-2,026.30 (year 7).

We next estimate net present value and durations of liability and asset cash flows. On the one hand, the NPV of the liability is $565,412.00, while NPV of asset cash flows is $565,410.00 for both PCA and ICA. On the other hand, in the PCA case, the durations for liability cash flows are -1.4267 (D1), -2.9026 (D2) and 1.9325 (D3), and for asset cash-flows, -1.4292 (D1), -2.8999

[3] McCulloch-Kochin (http://economics.sbs.ohio-state.edu/jhm/ts/ts.html).

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Maturity	Sep-07	PCA			ICA		
		U1	U2	U2	A1	A2	A3
0.083	3.963%	-0.0364	-0.1033	0.0856	-1.3401	-0.5829	0.0289
0.167	3.964%	-0.0368	-0.1026	0.0842	-1.3342	-0.5945	0.0183
0.25	3.965%	-0.0372	-0.1020	0.0829	-1.3278	-0.6056	0.0085
0.333	3.965%	-0.0376	-0.1014	0.0815	-1.3220	-0.6172	-0.0022
0.417	3.966%	-0.0380	-0.1007	0.0802	-1.3151	-0.6281	-0.0121
0.5	3.967%	-0.0384	-0.1000	0.0788	-1.3089	-0.6393	-0.0224
0.583	3.968%	-0.0388	-0.0994	0.0775	-1.3022	-0.6505	-0.0324
0.667	3.968%	-0.0392	-0.0987	0.0762	-1.2958	-0.6619	-0.0426
0.75	3.969%	-0.0396	-0.0980	0.0749	-1.2891	-0.6727	-0.0524
0.833	3.970%	-0.0400	-0.0974	0.0735	-1.2829	-0.6840	-0.0629
0.917	3.970%	-0.0404	-0.0967	0.0722	-1.2764	-0.6949	-0.0727
1	3.971%	-0.0408	-0.0961	0.0709	-1.2696	-0.7054	-0.0823
2	3.983%	-0.0452	-0.0878	0.0560	-1.1848	-0.8270	-0.1932
3	4.019%	-0.0490	-0.0794	0.0423	-1.0949	-0.9315	-0.2912
4	4.091%	-0.0520	-0.0710	0.0299	-1.0012	-1.0152	-0.3744
5	4.184%	-0.0539	-0.0626	0.0188	-0.9043	-1.0741	-0.4411
6	4.278%	-0.0546	-0.0547	0.0082	-0.8128	-1.1099	-0.4957
7	4.364%	-0.0543	-0.0476	-0.0017	-0.7301	-1.1265	-0.5403
8	4.443%	-0.0533	-0.0411	-0.0108	-0.6554	-1.1297	-0.5761
9	4.522%	-0.0522	-0.0352	-0.0192	-0.5861	-1.1271	-0.6065
10	4.605%	-0.0510	-0.0295	-0.0269	-0.5189	-1.1234	-0.6336
15	4.964%	-0.0469	-0.0048	-0.0518	-0.2101	-1.0868	-0.7050
20	4.964%	-0.0444	0.0148	-0.0436	0.0890	-0.9558	-0.5921
30	4.722%	-0.0419	0.0418	0.0212	0.6098	-0.5730	-0.1098
40	4.598%	-0.0406	0.0562	0.0584	0.8935	-0.3590	0.1632

Table 3. ICA Yields, Eigenvectors and Weights for September 2007

(D2) and 1.9347 (D3); while in the ICA case, liability durations are -3.8946 (D1), -2.5747 (D2) and -0.5335 (D3), and for asset they are the same, -3.8946 (D1), -2.5747 (D2) and -0.5335 (D3).

Table 4 shows liability, asset-PCA and asset-ICA cash flows present values, differences between the liability and each asset present values ($surplus) and the hedge ratio. Once again, only the PCA portfolio hedge ratio, now for a six-month planning period, is outside the IAS 39 range, reaching a value of 130.11%.

Date	Liability Value	Asset Value PCA	Asset Value ICA	Surplus($) PCA	Surplus($) ICA	Hedged(%) PCA	Hedged(%) ICA
Sep-07	565,412.00	565,410.00	565,410.00	-2.00	-2.00		
Oct-07	569,529.28	569,549.25	569,549.85	19.97	20.57	99.47%	99.45%
Nov-07	575,946.55	576,012.48	576,013.56	65.93	67.01	99.29%	99.28%
Dec-07	576,950.12	577,046.22	577,027.86	96.10	77.74	97.08%	98.94%
Jan-08	585,448.79	585,410.04	585,413.97	-38.75	-34.82	101.61%	101.34%
Feb-08	589,911.12	589,927.77	589,914.66	16.66	3.54	98.77%	99.15%
Mar-08	591,046.74	590,800.60	590,895.76	-246.14	-150.98	130.11%	115.75%
Apr-08	588,297.00	588,101.00	588,182.57	-196.00	-114.43	101.86%	101.35%
May-08	588,040.01	587,825.07	587,915.28	-214.94	-124.74	93.13%	96.14%
Jun-08	589,733.20	589,370.80	589,524.53	-362.40	-208.67	109.54%	105.22%
Jul-08	591,343.56	591,010.97	591,150.13	-332.59	-193.43	98.18%	99.06%
Aug-08	592,448.88	592,140.12	592,262.27	-308.76	-186.61	97.89%	99.39%
Sep-08	594,367.64	594,079.72	594,199.99	-287.93	-167.65	98.93%	99.02%

Table 4. Immunization Results (Oct-07 to Sep-08)

Finally, we summarize the performance of the two alternative techniques to immunize the

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portfolio, computing the mean of surplus for the twelve planning periods in the two test periods. In this sense we also compute the standard deviation around the null surplus and their extreme values. Table 5 shows a smaller range for the interval of CityICA surplus versus PCA surplus, and a standard deviation around zero quite smaller for the ICA technique.

Statistics	Feb-91 to Feb-92		Sep-07 to Sep-08	
	Surplus($) PCA	Surplus($) ICA	Surplus($) PCA	Surplus($) ICA
mean	23.64	-45.92	-137.76	-78.04
std. Dev.	253.45	148.51	220.07	131.51
max	424.69	252.62	96.10	77.74
min	-429.19	-320.19	-362.40	-208.67

Table 5. Performance of the two alternative techniques to immunize

5. CONCLUSIONS

Principal Component Analysis is generally used in portfolio management to extract a reduced number of explanatory factors from the historical behavior of a set of variables. This, in turn, enables the use of parsimonious valuation models that are relatively easy to apply in risk management. With respect to fixed-income portfolio management, PCA has contributed to the design of immunization strategies in which the first three principal components explain a large part of term structure interest rate behavior.

However, the implicit Gaussian hypothesis in PCA contrasts with the leptokurtic behavior of financial variables such as interest rates. In this sense, other statistical techniques that do not incorporate this constraint could improve PCA results. In this article, we show how the use of Independent Component Analysis (ICA), which has seldom been used in portfolio management, may enhance the performance of financial immunization strategies.

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APPENDIX

Algorithm-1

1. Set $j = 1$.
2. Choose Ninitial vectors of parameters $w$of unit norm[4] $[W^{(j)} = \left(w_1^{(j)}, ..., w_N^{(j)}\right)^T]$.
3. Let $\forall i = 1, ..., N \quad w_i^{(j)} \leftarrow \mathrm{E}\left[z \cdot \psi_k'\left(w_i^T \cdot z\right)\right] - \mathrm{E}\left[\psi_k''\left(w_i^T \cdot z\right)\right] \cdot w_i^T$. Where $\psi_k'(\cdot)$ denotes first derivative and $\psi_k''(\cdot)$¨second of $\Psi(\cdot)$ function. In practice, the sample mean is applied for $\mathrm{E}(\cdot)$.
4. Symmetric orthogonalization: $W \leftarrow W \bullet \left(W^T \bullet W\right)^{-\frac{1}{2}}$, or Gram-Schmidt orthogonalization.
5. Normalization: $\forall i = 1, ..., N \quad w_i \leftarrow \frac{w_i}{\|w_i\|}$
6. If no converged[5], i.e. $\left\|W^{(j)} - W^{(j-1)}\right\| \neq 0$, go back to c.
7. Set $j = j + 1$. For $j < N \; or \; K$, go back to step b.

Algorithm-2

1. Set $j = 1$.
2. Choose Ninitial vectors of parameters $w$of unit norm $[W^{(j)} = \left(w_1^{(j)}, ..., w_N^{(j)}\right)^T]$.
3. Estimate: $U = Z^T \bullet W^{(j)}$.
4. Calculate: $\forall i = 1, ..., N$: $\beta_i = -\mathrm{E}\left[u_i \cdot \psi_1'(u_i)\right]$ and $\alpha_i = -\left[\beta_i + \mathrm{E}\left[\psi_1''(u_i)\right]\right]^{-1}$.
5. Let: $W^{(j)} \leftarrow \mu \cdot \left\{W^{(j)} + diag(\alpha_i) \bullet \left\{diag(\beta_i) + \mathrm{E}\left[\psi_k(U) \bullet U^T\right]\right\} \bullet W^{(j)}\right\}$. Where μ is the learning cost.
6. Symmetric orthogonalization: $W \leftarrow W \bullet \left(W^T \bullet W\right)^{-\frac{1}{2}}$, or Gram-Schmidt orthogonalization.
7. Normalization: $\forall i = 1, ..., N \quad w_i \leftarrow \frac{w_i}{\|w_i\|}$
8. If no converged, i.e. $\left\|W^{(j)} - W^{(j-1)}\right\| \neq 0$, go back to c.
9. Set $j = j + 1$. For $j < N \; or \; K$, go back to step b.

The first and second derivatives of the functions $\Psi(\cdot)$ are one of following;

$$\psi_1'\left(w^T z\right) = \tanh\left[\lambda_1\left(w^T z\right)\right] \qquad \psi_1''(z) = \lambda_1\left\{1 - \tanh\left[\lambda_1\left(w^T z\right)\right]\right\} \qquad \lambda_1 \in [1, 2]$$
$$\psi_2'\left(w^T z\right) = \left(w^T z\right)\exp\left(\frac{-\left(w^T z\right)^2}{2}\right) \qquad \psi_2''\left(w^T z\right) = \left[1 - \left(w^T z\right)^2\right]\exp\left(\frac{-\left(w^T z\right)^2}{2}\right)$$
$$\psi_3'\left(w^T z\right) = \left(w^T z\right)^3 \qquad \psi_3''\left(w^T z\right) = 3\left(w^T z\right)^2$$

[4]We recommend using the norm sum of absolute values.

[5]In this case we use Frobenius norm or root of sum square.

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