# Capital Structure: Optimal leverage and maturity choice in a dynamic model

**Santiago FORTE**[1]

**Abstract:** We introduce a model in which risk-free interest rate, firm risk, bankruptcy costs, issuance costs, tax benefits on debt, and earnings ratio, determine the optimal choice of leverage and maturity. The model assumes that debt pays a regular flow of interest, allows the firm to rebalance its optimal capital structure at maturity issuing new debt at par, links tax deductions to the presence of taxable income, and considers default to be an endogenous and time-dependent decision. Simulation results are also provided, with standard leverage ratios, debt maturities, and credit spreads being replicated for reasonable parameter values.

**Key words:** capital structure, optimal leverage, optimal maturity, dynamic model

**JEL Classification:** G32

**Resumen**: Presentamos un modelo en el que tipo libre de riesgo, riesgo de la empresa, costes de quiebra, costes de emisión, beneficios fiscales a la deuda, y ratio de beneficios, determinan la elección óptima de apalancamiento y vencimiento. El modelo asume que la deuda paga un flujo regular de intereses, permite que la empresa reajuste su estructura óptima al vencimiento emitiendo nueva deuda a la par, condiciona las deducciones fiscales a la presencia de beneficios fiscalmente imputables, y considera el impago una decisión óptima y dependiente del tiempo. Se ofrecen asimismo los resultados de diversas simulaciones donde niveles habituales de apalancamiento, vencimiento, y primas de crédito, son generados a partir de valores razonables de los parámetros.

**Título**: Estructura de Capital: Apalancamiento y Vencimiento Óptimos en un Modelo Dinámico

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1 ESADE Business School - Ramon Llull University. Av. Torreblanca 59, 08172, Sant Cugat del Vallès, Barcelona (Spain). Tel: +34 932 806 162. Fax: +34 932 048 105. santiago.forte@esade.edu. Forte acknowledges financial support from MEC Grant Ref: AP2000-1327, and thanks J. Ignacio Peña, Santiago Velilla, Ramón Xifré and Isabel Abínzano, for helpful suggestions. He is especially grateful to the members of the RiskLab Toronto, and the Department of Mathematics of the University of Toronto, for their personal and material contribution in the development of simulations contained in this document. The usual disclaimers apply.



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## 1.- INTRODUCTION

We introduce a model in which risk-free interest rate, firm risk, bankruptcy costs, issuance costs, tax benefits on debt, and earnings ratio, determine the optimal choice of leverage and maturity. The model, which is built on the theoretical framework originated by Kane, Marcus and McDonald (1985) in a zero coupon bond model, and followed by Fischer, Heinkel and Zechner (1989) in a perpetual debt model, assumes that debt pays a regular flow of interest, allows the firm to rebalance its optimal capital structure at maturity issuing new debt at par, links tax deductions to the presence of taxable income, and considers default to be an endogenous decision: at every period, equity holders decide whether or not they are willing to finance interest payments (or interest payments plus principal at maturity). Unlike previous models, this optimal decision is assumed to be time dependent.[2] Simulation results are also provided, with standard leverage ratios, debt maturities and credit spreads being replicated for reasonable parameter values.

Our model should be related to recent work by Ju and Ou-Yang (2006), who analyze the simultaneous choice of leverage and maturity in a model with stochastic interest rates.[3] While their work presents the interesting novelty of allowing the risk-free rate to be stochastic within an optimal capital structure model, the added complexity does not seem to produce substantial differences compared with a model with constant interest rate. In their own words:[4]

> *"Our calculations indicate that the results with stochastic interest rates are similar to a model with a constant interest rate whose value is calibrated to the long-run mean of the stochastic interest rate. Thus, our model provides a justification for the use of constant interest rates in capital structure models, provided that the long-run mean of the interest rate process instead of the spot interest rate or a YTM* (yield to maturity) *is used as the input for the constant interest rate."*

Comparative statics analysis indicates on the other hand that the Ju and Ou-Yang (2006) model replicates standard leverage ratios fairly well for the selected parameter values. However, those same parameters imply optimal debt maturities and credit spreads which are below commonly observed levels. We find in this paper that this is not the case in a model where (contrary to Ju and Ou-Yang (2006)) tax deductions are subject to the presence of positive profits. What makes the difference is that by imposing this restriction, the effect of a higher risk-free rate over the optimal leverage and the final credit spread reverts from positive to negative, while the effect over optimal maturity remains negative. The prediction of a positive relation between risk-free rate and leverage ratio in Ju and Ou-Yang (2006) relies mainly on the traditional argument of a higher risk-neutral drift in the asset value, which would allow the

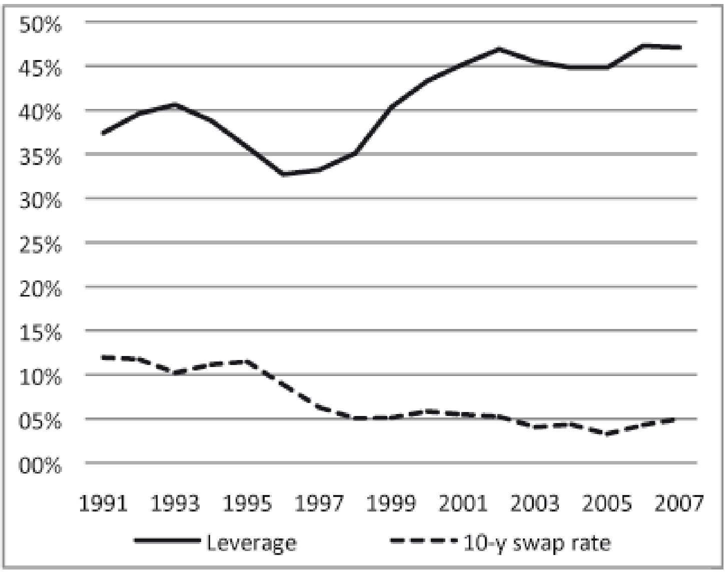
2 It is noteworthy that several factors different from the interest of stockholders- i.e. debt covenants (Leland (1994)), liquidity restrictions (Davydenko (2005)), insolvency codes (Franks, Nyborg and Torous (1996)) - may, at some point, trigger default. However, Alonso, Forte and Marqués (2008) document a strong, positive relation, between the endogenous default barrier and the CDS implied default barrier. As a result, considering the endogenous default point seems more suitable than imposing an exogenous (probably arbitrary) default point.

3 Many aspects of the present paper can also be found in Leland and Toft (1996). However, debt maturity is exogenously rather than endogenously determined in their model

4 Ju and Ou-Yang (2006, p. 16)

firm to issue more debt in order to increase the coupon, and therefore, tax deductions. On the contrary, when these benefits are linked to the presence of taxable income, a higher coupon makes tax benefits less likely given the higher probability of falling into losses. The firm in this case reduces leverage in order to control this risk. Because the default risk is shortened due to lower leverage and lower maturity, the credit spread falls as well. A negative relation between interest rates and debt levels seems not only more in line with economic intuition, but consistent with empirical evidence: Graham and Harvey (2001) find that managers prefer to issue debt when interest rates are lower, while Antoniou, Guney and Paudyal (2008) document a negative relation between the term structure of interest rates and the leverage ratio. Furthermore, the recent expansion in debt levels in a context of decresing interest rates seems to point in the same direction. Figure 1 reflects both the evolution in the 10 year swap rate and the levarge ratio in Spain during the period 1991-2007. It is apparent how both series have moved in opposite directions.

**Figure 1**




This figure shows the evolution in the 10 year swap rate in Spain during the period 1991-2007 (source: Datastream), together with the evolution in the leverage ratio (source: Central de Balances, Banco de España).

The rest of the article is organized as follows: Section 2 presents the model; formal proofs are left to the Appendix. Section 3 develops the numerical algorithm that solves the optimization problem, and provides simulation results. Finally, Section 4 offers some concluding remarks.

## 2. THE MODEL

We assume that the value of the firm's unlevered assets evolves according to the diffusion process

$$dA = \mu A dt + \sigma A dz \,, \qquad (1)$$

where $\mu$ is the expected rate of return on $A$, $\sigma$ is the volatility of the rate of return which is assumed to be constant, and $dz$ is the increment of a Wiener process.

Consider that the firm issues a coupon bond with principal $P$ and maturity in $T$ years. To keep the analysis as simple as possible, we restrict $T$ to a natural number, and coupon payments, denoted by $c$, to be concentrated at the end of every year. Debt issuance generates a cost of $\beta P$ which is borne by equity holders, with $\beta \in (0,1)$. All coupon payments, and the final payment of coupon plus principal, are to be financed by issuing additional equity. Tax deductions, $\tau$, on interest payments are assumed. We also assume that these deductions apply as long as the firm is profitable. If the firm's annual EBIT represents a constant ratio, $\varepsilon$, of the firm's assets value, then this assumption implies that tax deductions are available in a given period if, and only if, $\varepsilon A > c$: that is, if and only if $A > Ad$, where $Ad = \frac{c}{\varepsilon}$. Finally, if the firm goes bankrupt, its assets, $A$, lose a fraction $\alpha \in [0,1]$ of their value due to bankruptcy costs.[5]

We denote $S(A,t)$, $D(A,t)$ and $v(A,t)$, the equity, debt and firm value respectively, when $t$ periods remain to maturity. Given that $A$ represents the market value of the firm's assets, it must reflect all possible future revenues arising from those assets, including tax benefits net of default costs when the firm is optimally levered. As a consequence, we may identify the market value of an unlevered firm with that of an optimally levered firm. However, as long as the firm is unlevered, it loses the return coming from tax benefits net of default costs. In other words, the unlevered firm earns a below-equilibrium rate of return. Following McDonald and Siegel (1984) and Kane, Marcus and McDonald (1985), we have that under the assumptions found in Merton's (1973) intertemporal CAPM, any contingent claim $F$, with underlying asset $A$, must satisfy the partial differential equation

$$\frac{1}{2}\sigma^2 A^2 F_{AA} + (r-\delta) A F_A - F_t - rF = 0 \,,$$

where $\delta$ represents precisely the difference between the equilibrium rate of return $\mu^*$ (necessary to compensate investors for bearing the risk of asset $A$) and the actual rate of return $\mu$, that is, it represents the difference in return between an unlevered firm and an optimally levered firm. As in Kane, Marcus and McDonald (1985), $\delta$ represents the measure of the tax benefits on debt net of default costs. $\delta$ affects the valuation of $S(A,t)$, $D(A,t)$ and $v(A,t)$ in the same way as a dividend yield.

The firm value is the sum of the equity value and the debt value. However, we may also take the firm value as the sum of two different assets. Specifically

$$v(A,t) = V(A,t) + TB(A,t),$$

$$t = 0, 1, \ldots, T-1, T^* \,,$$

5 Although default need not lead to bankruptcy, we will use the terms default and bankruptcy interchangeably to describe the situation in which the firm defaults. This may in fact lead to any situation from informal or formal restructuring to formal liquidation. What happens when a firm defaults is not modeled here. Note also that the presence of bankruptcy costs means that equation (1) does not hold when the firm defaults.

where $TB(A,t)$ represents the present value of the tax benefits on debt. $V(A,t)$ on the other hand, describes an asset that generates the firm assets value whenever it becomes unlevered, either because it defaults and falls into debt holders' hands, or because the debt finally matures. It does not coincide with the current value of the firm assets for two reasons: first, $\delta$ affects the present value of these assets, in the same way the value of a forward contract (with zero delivery price) is affected when the underlying asset pays a dividend yield. Second, if the firm becomes unlevered because it defaults, then the value of those assets loses a rate $\alpha$ due to bankruptcy costs.[6] Note that we define $v(A,t)$ as the sum of these two components from $t=0$ to $t=T^+$. This is to differentiate the firm value after issuance, $v^+(A,t)$, from the firm value before issuance, $v^-(A,t)$. In this latter case, another component must be subtracted: the issuance costs $\beta P$. Then

$$v^-(A,T)=v^+(A,T)-\beta P\,. \tag{2}$$

Assuming for the moment that the bankruptcy-triggering firm assets value when $t$ periods remain to maturity, $Ab_t$, is exogenous and strictly lower than $Ad$, it is possible to show that[7]

$$\begin{aligned} S^-(A,T) &= S^+(A,T)-\beta P \\ &= Ae^{-\delta T}N_T(\boldsymbol{a}_{T,0})-Pe^{-rT}N_T(\boldsymbol{b}_{T,0}) \\ &\quad -\sum_{k=0}^{T-1} ce^{-r(T-k)}\left[N_{T-k}(\boldsymbol{b}_{T,k})-\tau N_{T-k}(\boldsymbol{c}_{T,k})\right]-\beta P, \end{aligned} \tag{3}$$

$$V(A,T)=Ae^{-\delta T}N_T(\boldsymbol{a}_{T,0})+(1-\alpha)A\sum_{k=0}^{T-1}e^{-\delta(T-k)}\left[N_{T-(k+1)}(\boldsymbol{a}_{T,k+1})-N_{T-k}(\boldsymbol{a}_{T,k})\right], \tag{4}$$

$$TB(A,T)=\sum_{k=0}^{T-1}\tau ce^{-r(T-k)}N_{T-k}(\boldsymbol{c}_{T,k}), \tag{5}$$

$$v^-(A,T)=V(A,T)+TB(A,T)-\beta P, \tag{6}$$

6 It would be possible to think of two different assets to represent these two effects. However, the mathematical exposition is simplified by considering both together.

7 Formal proof can be found in the Appendix.

$$D(A,T)=v^{-}(A,T)-S^{-}(A,T)=v^{+}(A,T)-S^{+}(A,T)$$

$$=(1-\alpha)A\sum_{k=0}^{T-1}e^{-\delta(T-k)}\left[N_{T-(k+1)}\left(\boldsymbol{a}_{T,k+1}\right)-N_{T-k}\left(\boldsymbol{a}_{T,k}\right)\right] \qquad (7)$$

$$+Pe^{-rT}N_{T}\left(\boldsymbol{b}_{T,0}\right)+\sum_{k=0}^{T-1}ce^{-r(T-k)}N_{T-k}\left(\boldsymbol{b}_{T,k}\right),$$

where $N_0(\cdot)=1$. For the more general case of $z\geq 1$, $N_z(\cdot)$ denotes the multivariate normal cumulative distribution function of dimension $z$, with marginal distribution for each component $N_1(0,1)$, correlation matrix $R_z=\{\rho_{ij}^{z}\}$, with[8]

$$\rho_{ij}^{z}=\begin{cases}\sqrt{\frac{z-i+1}{z-j+1}} & if \quad i\geq j\\ \sqrt{\frac{z-j+1}{z-i+1}} & if \quad i<j,\end{cases}$$

and integration limits[9]

$$\begin{aligned}\boldsymbol{a}_{t,k}&=\begin{bmatrix}a_{t,k} & a_{t,k+1} & \dots & a_{t,t-1}\end{bmatrix},\\ \boldsymbol{b}_{t,k}&=\begin{bmatrix}b_{t,k} & b_{t,k+1} & \dots & b_{t,t-1}\end{bmatrix},\\ \boldsymbol{c}_{t,k}&=\begin{bmatrix}c_{t,k} & b_{t,k+1} & \dots & b_{t,t-1}\end{bmatrix},\end{aligned}$$

$$t=1,\dots,T,$$

$$k=0,\dots,t-1,$$

where

$$a_{t,s}=\frac{\ln\left(\frac{A_T}{Ab_s}\right)+\left(r-\delta+\frac{\sigma^2}{2}\right)(t-s)}{\sigma\sqrt{t-s}},$$

8 Although this extension of Kane, Marcus and McDonald (1985) to a coupon bond is based on the methodology developed by Geske (1977) to value corporate liabilities as compound options, the correlation matrix we provide apparently differs from that derived by Geske (1977). This is a simple question of which is the 'first', 'second', ..., and 'last' variable in the multivariate normal density function. As an example, the element $\rho_{12}^{z}$ in Geske (1977) would be $\sqrt{\frac{1}{2}}$, while this is the value of the element $\rho_{(z-1)z}^{z}$ in our case.

9 $a_{t,k}$ is the integration limit of the 'first' variable in the multivariate normal density function, while $a_{t,t-1}$ that of the 'last' variable.

$$b_{t,s} = a_{t,s} - \sigma\sqrt{t-s},$$

$$c_{t,s} = \frac{\ln\left(\frac{A_t}{Ad}\right) + \left(r - \delta - \frac{\sigma^2}{2}\right)(t-s)}{\sigma\sqrt{t-s}},$$

$$s = k, \ldots, t-1,$$

with $A_t$ denoting the firm assets value when $t$ periods remain to maturity.

We may now derive the endogenous bankruptcy-triggering threshold, $Ab_t$, as the $A$ value below which equity holders choose not to pay when $t$ periods remain to maturity. It is reasonable to assume that the firm assets value making the firm default is lower than that making it lose tax deductions.[10] The consequence is that we may assume that the payment that equity holders choose not to satisfy, bringing about default, is $P + c$ and not $P + c(1-\tau)$ at maturity, and $c$ and not $c(1-\tau)$ at any other period. In order to derive the $T$ critical threshold values, we should proceed recursively, that is, we can first find $Ab_0$ as the solution to

$$S(A,0) = A - P - c = 0,$$

getting, $Ab_0 = P + c$, and then find $Ab_1$, $Ab_2$, …, $Ab_{T-1}$ sequentially as the implicit solution to

$$S(A,t) = Ae^{-\delta t}N_t(a_{t,0}) - Pe^{-rt}N_t(b_{t,0}) - \sum_{k=0}^{t-1} ce^{-r(t-k)}\left[N_{t-k}(b_{t,k}) - \tau N_{t-k}(c_{t,k})\right] - c = 0, \quad (8)$$

for $t = 1, \ldots, T-1$.

## 3. OPTIMAL CAPITAL STRUCTURE

The capital structure should be chosen so as to maximize the market value of the firm, as a function of the market value of the firm's unlevered assets. Given that we assume that the market value of the firm's unlevered assets fully reflects the possibility of an optimal leverage, the best the firm can do is confirm market expectations. In other words, if the firm is unlevered, its market value should reflect its implicit option to choose an optimal leverage. If the firm is non-optimally levered later on, its value will fall, reflecting its constraint to a non-optimal leverage for a given period of time. As a result, the solution to the optimization problem[11]

$$Max_{P,T} \quad v^{-}(A,P,T)$$

$$\text{s.t.} \quad D(A,P,T) = P, \quad (9)$$

should be that

$$v^{-}(A,P^{*},T^{*}) = A. \quad (10)$$

10 Typically, default is preceded by a long period of continuous losses in which tax deductions are not enjoyed.

11 The restriction states that debt should be issued at par. Note also that it is assumed that the same problem will be solved at maturity, making the model dynamic by construction (see Kane, Marcus and McDonald (1985)).

But maximizing the firm value is equivalent to maximizing the return arising from the tax advantage to debt, which is measured by $\delta$. Therefore, no pair $(P,T)$ that solves (9) and results in (10) for a given $\delta$ can be optimal, if there is another pair $(P^*,T^*)$ that solves (9) and results in (10) for a higher $\delta$. The solution will then be given by a vector $(P^*,T^*,\delta^*)$ whereby $\delta^*$ is the highest possible $\delta$ for which the solution to (9) is a pair $(P^*,T^*)$, which in addition makes condition (10) hold. We then propose the following numerical algorithm:[12]

1. Set $A=100$, and some initial $T_0$ and $\delta_0$.[13]

2. Search the $P$ value that maximizes $v^-$, given $T_0$ and $\delta_0$. This requires the following procedure: For each proposal for $P$, search the $c$ value that makes $D=P$. Each proposal for $c$ as the solution to $D=P$ implies at the same time computing $Ab_0$, $Ab_1$, …, $Ab_{T-1}$ sequentially as described in Section 2.

3. Once the optimal $P$ has been found, check whether $v^- = A$.

4. If $v^- \neq A$, find a new $\delta_1$ such that $v^- = A$, given $P$. Again, each proposal for $\delta_1$ implies searching for the $c$ value that makes $D=P$, and each proposal for $c$ implies recomputing $Ab_0$, $Ab_1$, …, $Ab_{T-1}$ values.

5. Using this new $\delta_1$ instead of $\delta_0$, go back to Step 2 and repeat until convergence, that is, until the resulting $v^-$ equals $A$ in Step 3.

6. Consider different $T$ values and search for the one that generates the maximum $\delta$.

The resulting vector $(P^*,T^*,\delta^*)$ is simultaneously consistent with the value maximizing criterion (Steps 2 and 6), and with conditions $v^- = A$ (Step 3) and $D=P$ (Steps 2 and 4). Note that the algorithm requires evaluating multivariate normal cumulative distribution functions of order equal to and lower than $T$. We can approximate these estimations by using Monte Carlo simulations. As an example, consider that we need to evaluate $N_2(\boldsymbol{a}_{2,0})$ for some given $a_{2,0}$ and $a_{2,1}$. In this case we generate 100,000 random observations from a bivariate normal density function, with marginal distribution for each component $N_1(0,1)$, and correlation matrix $R_2$. The result is 100,000 pairs $(\xi_1,\xi_2)$. We then compute the number of simultaneous occurrences of $\xi_1 < a_{2,0}$ *and* $\xi_2 < a_{2,1}$. The ratio of this number of *favorable cases* over the total number of *possible cases* finally gives us an approximation of $N_2(\boldsymbol{a}_{2,0})$. Base case parameters are chosen as follows:

$$r = 0.04$$
$$\sigma = 0.25$$
$$\alpha = 0.15$$
$$\beta = 0.01$$
$$\tau = 0.25$$
$$\varepsilon = 0.035.$$

12 Although this is similar in spirit to that in Kane, Marcus and Mc-Donald (1985), there are two main differences: first, we have to compute $T$ bankruptcy-triggering firm asset values that are not present in their model. Second, we require debt to be issued at par, which they do not need to do given their different formulation of the problem.

13 $A$ can be arbitrarily fixed, given that the model does not include scale effects.

Ibbotson Associates (1997) reports a historical interest rate on U.S. Treasury Bills of around 0.037. The standard deviation of the value of unlevered assets is the same used by Kane, Marcus and McDonald (1985), and by Fisher, Heinkel and Zechner (1989), and similar to the one applied in other models. Bankruptcy costs are the mean of the range found by Andrade and Kaplan (1998), who estimate financial distress costs to be around 10 to 20% of firm value. Issuance costs are consistent with estimations provided by Blackwell and Kidwell (1988). They find flotation costs to represent 1.165% of the issue size for public issues and 0.795% for private issues. Tax advantage to debt is chosen to represent not only corporate but also personal taxes (Miller, 1977). Finally, the EBIT ratio generates a 'price-earnings ratio' for an optimally levered firm equal to 16.25,[14] close to its historical average, which is around 17.

We provide simulation results in Tables 1 and 2. In the base case the optimal leverage is 43.10%. Rajan and Zingales (1995) find non-equity liabilities to represent on average 44% of total assets for U.S. firms. Optimal maturity at issuance is 6 years. This is consistent with an average debt maturity of about 3 years as reported by Stohs and Mauer (1996). The tax advantage to debt is 25.8 basis points. In this, and in the other cases, the advantage to debt appears higher than predicted by the one-period model in Kane, Marcus and McDonald (1985). The model is also consistent in predicting firm values that trigger loss of tax deductions always higher than those that trigger default. Interestingly, the endogenous bankruptcy-triggering firm value is always monotonically increasing, a feature sometimes imposed in models with exogenous default barrier (Black and Cox (1979) and Ju, Parrino, Poteshman and Weisbach (2005)). The credit spread for the base case is 51.94 basis points. Leland (1994) argues that the historical credit spread of investment-grade bonds with no call provision would be around 52 basis points.

**Table 1. Comparative Statics**

| | *Optimal Leverage* | *Coupon* | *Credit Spread* | *Optimal Maturity* | *Tax Advantage* | *Ad* |
|---|---|---|---|---|---|---|
| *Base Case* | 43.10 | 1.99 | 51.94 | 6 | 25.80 | 56.93 |
| $r = 0.06$ | 37.94 | 2.42 | 17.65 | 4 | 34.33 | 69.06 |
| $\sigma = 0.1$ | 68.65 | 2.85 | 6.93 | 8 | 46.67 | 81.46 |
| $\alpha = 0.05$ | 51.10 | 2.46 | 70.63 | 7 | 31.26 | 70.35 |
| $\beta = 0.015$ | 42.57 | 1.97 | 51.99 | 8 | 23.11 | 56.24 |
| $\tau = 0.45$ | 45.18 | 2.28 | 68.92 | 5 | 59.41 | 65.06 |
| $\varepsilon = 0.04$ | 48.15 | 2.33 | 73.24 | 8 | 28.00 | 58.34 |

Optimal leverage (%), coupon, credit spread (b.p), optimal maturity (years), tax advantage to debt (b.p), and zero tax benefits-triggering firm value (*Ad*), for different parameter values.



14 This is computed as the ratio $\frac{S}{\varepsilon A}$. Note that even though coupon payments are taken into account for taxation purposes, they actually do not alter earnings per share given that they are already included in the equity price.

**Table 2. Comparative Statics (cont.)**

| | $Ab_{T-1}$ | $Ab_{T-2}$ | $Ab_{T-3}$ | $Ab_{T-4}$ | $Ab_{T-5}$ | $Ab_{T-6}$ | $Ab_{T-7}$ | $Ab_{T-8}$ |
|---|---|---|---|---|---|---|---|---|
| *Base Case* | 33.95 | 34.37 | 35.09 | 36.19 | 38.00 | 45.10 | - | - |
| $r$ = 0.06 | 33.26 | 34.03 | 35.35 | 40.35 | - | - | - | - |
| $\sigma$ = 0.1 | 67.39 | 67.54 | 67.70 | 67.97 | 68.24 | 68.48 | 69.18 | 71.50 |
| $\alpha$ = 0.05 | 40.70 | 41.08 | 41.51 | 42.25 | 43.58 | 45.44 | 53.56 | - |
| $\beta$ = 0.015 | 32.75 | 33.12 | 33.49 | 33.92 | 34.63 | 35.69 | 37.52 | 44.54 |
| $\tau$ = 0.45 | 38.55 | 39.28 | 40.44 | 42.21 | 49.71 | - | - | - |
| $\varepsilon$ = 0.04 | 37.70 | 38.08 | 38.43 | 38.86 | 39.62 | 40.88 | 42.75 | 50.49 |

Bankruptcy-triggering firm value when $k$ periods remain to maturity ( $Ab_k$ ), for different parameter values.

A higher risk-free interest rate seems to imply lower leverage. This is a reasonable result that optimal capital structure models have traditionally failed to generate. What distinguishes our model from those models is the recognition that tax benefits are lost when the firm incurs zero or negative profits. A higher risk-free interest rate means higher coupon payments and a greater stream of tax benefits. But higher coupon also means a lower probability of getting these benefits. This latter effect more than offsets the former and the firm reduces its leverage to reduce the coupon (which is still higher than in the base case), and with it, the probability of making losses. An increase in the risk-free rate also has the effect of reducing optimal debt maturity. Lower maturity could be interpreted as a policy complementing the reduction in leverage, that is, lower maturity also means a lower probability of making losses during the holding period. Both the reduction in leverage and maturity lead to a lower credit spread. Finally, the tax advantage to debt increases with the risk-free rate. This reflects the fact that 'buying tax benefits becomes cheaper', or in other words, the firm needs a lower principal to obtain the same coupon, and lower principal means lower issuance costs (even though these occur more frequently). It is noteworthy, on the other hand, that Ju and Ou-Yang (2006), assuming a (long-run) risk-free rate equal to 7%, report estimated values which are similar to those that arise in our model for a 6% rate. Although lowering the risk-free rate to 4% also improves the optimal maturity prediction in their model, results in terms of the optimal leverage and the credit spread worsen. This is explained by that fact that their model predicts not a negative, but a positive relation between leverage ratio and risk-free rate.

Higher volatility of the firm's unlevered assets leads to a lower leverage and a lower debt maturity. The reduction in leverage is significant enough to reduce the coupon in spite of the firm's higher risk. Basically, the firm seeks to reduce the coupon to control the risk of incurring losses. Lower leverage implies lower issuance costs, while a higher risk results in higher benefits from allowing the firm to rebalance its leverage more often. Both of these effects bring a reduction in the maturity of the debt. The lower stream of tax benefits has a higher impact than the reduction in issuance costs and the result is a lower tax advantage to debt.

The fourth line in Table 1 analyzes the case of lower bankruptcy costs. These imply a higher leverage. Lower 'loss given default' allows the firm to incur a higher default probability to increase the stream of tax benefits. Higher leverage, on the other hand, induces the firm to increase debt maturity in order to face issuance costs less often. More leverage and longer maturity more than offsets the reduction in bankruptcy costs, and the result is a higher credit spread. The reduction in bankruptcy costs finally brings an increase in the tax advantage to debt.

The next line in Table 1 considers a higher issuance cost. Although we observe a reduction in the leverage ratio, this is not as large as one might assume. The impact on debt maturity is greater. Higher issuance costs lead the firm to reduce the frequency with which it faces these costs, and also to reduce the tax advantage to debt.

An increase in the relevant tax ratio on the other hand, leads to greater leverage, increasing the credit spread. This time, tax deductions are more valuable, but in principle less likely because of the higher coupon. To offset the negative effect of a higher coupon on the probability of getting tax benefits, the firm reduces the debt maturity. A higher tax rate implies the predictable result of a higher benefit from issuing debt.

Finally, the higher the earnings ratio, the higher the leverage and the debt maturity. This is reasonable because the firm can then face a higher coupon and a longer maturity while controlling the risk for non-positive profits. Of course, all of this implies higher tax benefits. A summary of main model predictions is provided in Table 3. For comparison we also include main predictions in Ju and Ou-Yang (2006).

**Table 3. Comparative of Models' Predictions**

| | *This Model* | | | | *Ju and ou-Yang (2006)* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Lev.* | *Mat.* | *CS* | *TA* | *Lev.* | *Mat.* | *CS* | *TA* |
| $r$ | – | – | – | + | + | – | + | + |
| $\sigma$ | – | – | + | – | – | – | + | – |
| $\alpha$ | – | – | – | – | – | – | – | – |
| $\beta$ | – | + | + | – | – | + | + | – |
| $\tau$ | + | – | + | + | + | – | + | + |
| $\varepsilon$ | + | + | + | + | / | / | / | / |

This table compares main model predictions with those in Ju and Ou-Yang (2006). Variables included are optimal Leverage (*Lev.*), optimal maturity (*Mat.*), credit spread (*CS*), and tax advantage to debt (*TA*).

## 4. CONCLUSIONS

We introduce in this paper a model in which risk-free interest rate, firm risk, bankruptcy costs, issuance costs, tax benefits on debt, and earnings ratio, determine the optimal choice of leverage and maturity. The model assumes that debt pays a regular flow of interest, allows the firm to rebalance its optimal capital structure at maturity issuing new debt at par, and considers default to be an endogenous decision: at every period, equity holders decide whether or not they are willing to finance debt obligations. This optimal decision is allowed to be time-dependent. One distinguishing aspect of the model is that it links tax deductions to the presence of taxable income. We argue that this explains the model's higher ability, as compared with previous studies, to replicate common leverage ratios, debt maturities, and credit spreads for standard parameter values.

### Appendix

We provide formal proof of the valuation formulas presented in the core of the article. We start by computing three fundamental multiple integrals. Take $0 \leq k < t \leq T$, where $k$, $t$ and $T$ are natural numbers, and define

$$G(t,k) = e^{-r(t-k)} \int_{Ab_{t-1}}^{\infty} \int_{Ab_{t-2}}^{\infty} \cdots \int_{Ab_k}^{\infty} A_k \prod_{h=k}^{t-1} f(A_h \mid A_{h+1}) dA_k \ldots dA_{t-2} dA_{t-1} ,$$

where $Ab_{t-1}$, $Ab_{t-2}$, ..., $Ab_k$ are for the moment some given values, and

$$f(A_h \mid A_{h+1}) = \frac{1}{\sqrt{2\pi}\sigma A_h} \exp\left\{ -\frac{1}{2} \frac{\left\{ \ln(A_h) - \left[ \ln(A_{h+1}) + \left( r - \delta - \frac{\sigma^2}{2} \right) \right] \right\}^2}{\sigma^2} \right\}$$

is the density function of $A_h$ conditional on $A_{h+1}$.

Consider the following change of variable:

$$\tilde{x}_l = \ln(A_l)$$
$$l = k, \ldots, t,$$

and define

$$f(\tilde{x}_h \mid \tilde{x}_{h+1}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{1}{2} \frac{\left\{ \tilde{x}_h - \left[ \tilde{x}_{h+1} + \left( r - \delta - \frac{\sigma^2}{2} \right) \right] \right\}^2}{\sigma^2} \right\}$$

Then

$$G(t,k) = e^{-r(t-k)} \int_{\ln(Ab_{t-1})}^{\infty} \int_{\ln(Ab_{t-2})}^{\infty} \cdots \int_{\ln(Ab_k)}^{\infty} e^{\tilde{x}_k} \prod_{h=k}^{t-1} f(\tilde{x}_h \mid \tilde{x}_{h+1}) d\tilde{x}_k \ldots d\tilde{x}_{t-2} d\tilde{x}_{t-1} .$$

Now use

$$\frac{\tilde{x}_h - \left[\tilde{x}_{h+1} + \left(r - \delta - \frac{\sigma^2}{2}\right)\right]}{\sigma} = \frac{\left\{\tilde{x}_h - \left[\tilde{x}_{h+1} + \left(r - \delta + \frac{\sigma^2}{2}\right)\right]\right\} + \sigma^2}{\sigma}$$

$$h = k, \ldots, t-1$$

to get

$$G(t,k) = A_t e^{-\delta(t-k)} \int_{\ln(Ab_{t-1})}^{\infty} \int_{\ln(Ab_{t-2})}^{\infty} \cdots \int_{\ln(Ab_k)}^{\infty} \frac{1}{(2\pi)^{(t-k)/2} \sigma^{t-k}}$$

$$\prod_{h=k}^{t-1} \exp\left\{-\frac{1}{2} \frac{\left\{\tilde{x}_h - \left[\tilde{x}_{h+1} + \left(r - \delta + \frac{\sigma^2}{2}\right)\right]\right\}^2}{\sigma^2}\right\} d\tilde{x}_k \ldots d\tilde{x}_{t-2} d\tilde{x}_{t-1} \,.$$

We can make an additional change of variable

$$x_l = \frac{\tilde{x}_l - \left[\ln(A_t) + \left(r - \delta + \frac{\sigma^2}{2}\right)(t-l)\right]}{\sigma\sqrt{t-l}}$$

$$l = k, \ldots, t-1.$$

Noting that

$$\frac{\tilde{x}_h - \left[\tilde{x}_{h+1} + \left(r - \delta + \frac{\sigma^2}{2}\right)\right]}{\sigma} = x_h\sqrt{t-h} - x_{h+1}\sqrt{t-h-1}$$

$$h = k, \ldots, t-2\,,$$

we may express $G(t,k)$ as

$$G(t,k) = A_t e^{-\delta(t-k)} \int_{-a_{t,t-1}}^{\infty} \int_{-a_{t,t-2}}^{\infty} \cdots \int_{-a_{t,k}}^{\infty} \frac{\sqrt{(t-k)!}}{(2\pi)^{(t-k)/2}}$$

$$\prod_{h=k}^{t-2} \exp\left\{-\frac{1}{2}\left[x_h\sqrt{t-h} - x_{h+1}\sqrt{t-h-1}\right]^2\right\} \exp\left\{-\frac{1}{2}x_{t-1}^2\right\} dx_k \ldots dx_{t-2} dx_{t-1} \,.$$



Now define $Q_z = \{q_{ij}^z\}$, as the square symmetric matrix of dimension $z$, where

$$q_{11}^{z} = z$$

$$q_{ii}^{z} = 2(z - i + 1) \quad for \quad i = 2, ..., z$$

$$q_{ij}^{z} = \begin{cases} -\sqrt{z-i+1}\sqrt{z-j+1} & if \quad |i-j| = 1 \\ 0 & if \quad |i-j| \geq 2 \end{cases}$$

It is possible to show that $Q_z = R_z^{-1}$. To see this, define $W_z = R_z Q_z = \{w_{ij}^z\}$, and consider the following cases:

a) $j = 1$

In this case it is easy to see that

$$w_{11}^{z} = \sum_{k=1}^{z} \rho_{1k}^{z} q_{k1}^{z} = \rho_{11}^{z} q_{11}^{z} + \rho_{12}^{z} q_{21}^{z} = 1$$

$$w_{i1}^{z} = \sum_{k=1}^{z} \rho_{ik}^{z} q_{k1}^{z} = \rho_{i1}^{z} q_{11}^{z} + \rho_{i2}^{z} q_{21}^{z} = 0 \quad for \quad i = 2, ..., z$$

b) $j = z$

Now check that

$$w_{iz}^{z} = \sum_{k=1}^{z} \rho_{ik}^{z} q_{kz}^{z} = \rho_{i(z-1)}^{z} q_{(z-1)z}^{z} + \rho_{iz}^{z} q_{zz}^{z} = 0 \quad for \quad i = 1, ..., z-1$$

$$w_{zz}^{z} = \sum_{k=1}^{z} \rho_{zk}^{z} q_{kz}^{z} = \rho_{z(z-1)}^{z} q_{(z-1)z}^{z} + \rho_{zz}^{z} q_{zz}^{z} = 1$$

c) $1 < j < z$

In this case

$$w_{ij}^{z} = \sum_{k=1}^{z} \rho_{ik}^{z} q_{kj}^{z} = \rho_{i(j-1)}^{z} q_{(j-1)j}^{z} + \rho_{ij}^{z} q_{jj}^{z} + \rho_{i(j+1)}^{z} q_{(j+1)j}^{z}$$

Consider the three possible situations, namely $i < i-1$, $i = j$, and $i \geq j+1$, and make straight-forward computations to see that $w_{ii}^{z} = 1$ and $w_{ij}^{z} = 0$ for $i \neq j$; $i = 1, ..., z$; $j = 2, ..., z-1$.

We can now use previous arguments, and $|R_z| = \frac{1}{z!}$, to express $G(t,k)$ as

$$G(t,k) = A_t e^{-\delta(t-k)} \int_{-a_{t,t-1}}^{\infty} \int_{-a_{t,t-2}}^{\infty} \cdots \int_{-a_{t,k}}^{\infty} \frac{1}{(2\pi)^{(t-k)/2} |R_{t-k}|^{1/2}} \exp\left\{-\frac{1}{2} X R_{t-k}^{-1} X'\right\} dx_k \ldots dx_{t-2} dx_{t-1},$$

where $X = [x_k \ \ldots \ x_{t-2} \ \ x_{t-1}]$. Finally

$$G(t,k) = A_t e^{-\delta(t-k)} N_{t-k}(\boldsymbol{a}_{t,k})$$

$$t = 1, ..., T$$

$$k = 0, ..., t-1$$

Also define

$$\hat{G}(t,k) = e^{-r(t-k)} \int_{Ab_{t-1}}^{\infty} \int_{Ab_{t-2}}^{\infty} \cdots \int_{0}^{Ab_k} A_k \prod_{h=k}^{t-1} f(A_h \mid A_{h+1}) dA_k \ldots dA_{t-2} dA_{t-1},$$

Previous derivations imply that

$$\hat{G}(t,k) = A_t e^{-\delta(t-k)} \int_{-a_{t,t-1}}^{\infty} \int_{-a_{t,t-2}}^{\infty} \cdots \int_{-\infty}^{-a_{t,k}} \frac{1}{(2\pi)^{(t-k)/2} |R_{t-k}|^{1/2}} \exp\left\{-\frac{1}{2} X R_{t-k}^{-1} X'\right\} dx_k \ldots dx_{t-2} dx_{t-1},$$

and finally

$$\hat{G}(t,k) = A_t e^{-\delta(t-k)} \left[N_{t-(k+1)}(\boldsymbol{a}_{t,k+1}) - N_{t-k}(\boldsymbol{a}_{t,k})\right]$$

$$t = 1, ..., T$$

$$k = 0, ..., t-1$$

Another useful multiple integral is the following:

$$H(t,k) = e^{-r(t-k)} \int_{Ab_{t-1}}^{\infty} \int_{Ab_{t-2}}^{\infty} \cdots \int_{Ab_k}^{\infty} \prod_{h=k}^{t-1} f(A_h \mid A_{h+1}) dA_k \ldots dA_{t-2} dA_{t-1},$$

Define

$$y_l = \frac{\ln(A_l) - \left[\ln(A_t) + \left(r - \delta - \frac{\sigma^2}{2}\right)(t-l)\right]}{\sigma\sqrt{t-l}}$$

$$l = k, ..., t-1$$

Then

$$\frac{\ln(A_h)-\left[\ln(A_{h+1})+\left(r-\delta-\frac{\sigma^2}{2}\right)\right]}{\sigma}=y_h\sqrt{t-h}-y_{h+1}\sqrt{t-h-1}$$

$$h=k,\ ...,\ t-2$$

and $H(t,k)$ reduces to

$$H(t,k)=e^{-r(t-k)}\int_{-b_{t,t-1}}^{\infty}\int_{-b_{t,t-2}}^{\infty}\cdots\int_{-b_{t,k}}^{\infty}\frac{1}{(2\pi)^{(t-k)/2}|R_{t-k}|^{1/2}}\exp\left\{-\frac{1}{2}YR_{t-k}^{-1}Y'\right\}dy_k...dy_{t-2}dy_{t-1}$$

where $Y=[y_k\ \dots\ y_{t-2}\ \ y_{t-1}]$. If we also set $H(t,k)=1$ for $t=k$, then the result is that

$$H(t,k)=e^{-r(t-k)}N_{t-k}(\boldsymbol{b}_{t,k})$$

$$t=1,\ ...,\ T$$

$$k=0,\ ...,\ t$$

A final multiple integral we will use is

$$I(t,k)=\begin{cases} e^{-r}\int_{Ad}^{\infty}f(A_k|A_{k+1})dA_k & if \quad t-k=1 \\ \\ e^{-r(t-k)}\int_{Ab_{t-1}}^{\infty}\cdots\int_{Ab_{k+1}}^{\infty}\int_{Ad}^{\infty}\prod_{h=k}^{t-1}f(A_h\mid A_{h+1})dA_k dA_{k+1}...dA_{t-1} & if \quad t-k\geq 2 \end{cases}$$

If we now set $I(t,k)=1$ for $t=k$, then previous arguments result in

$$I(t,k)=e^{-r(t-k)}N_{t-k}(\boldsymbol{c}_{t,k})$$

$$t=1,\ ...,\ T$$

$$k=0,\ ...,\ t$$

At this point we are ready to derive specific expressions for equity, debt and firm value.

### Equity:

The equity value when the debt matures is given by

$$S(A,0)=\begin{cases} A-P-(1-\tau)c & if \quad A>Ad \\ \\ Max\{0, A-P-c\} & if \quad A\le Ad \end{cases}$$

and the firm defaults whenever $A_0 \le Ab_0 = P + c$ .

When only one year remains to maturity

$$S(A,1)=\begin{cases} G(1,0)-PH(1,0)-\sum_{k=0}^{1} c[H(1,k)-\tau I(1,k)] & if \quad A>Ad \\ \\ Max\{0, G(1,0)-PH(1,0)-c[H(1,0)-\tau I(1,0)]-c\} & if \quad A\le Ad \end{cases}$$

$$=\begin{cases} Ae^{-\delta}N_1(\boldsymbol{a}_{1,0})-Pe^{-r}N_1(\boldsymbol{b}_{1,0})-\sum_{k=0}^{1} ce^{-r(1-k)}[N_{1-k}(\boldsymbol{b}_{1,k})-\tau N_{1-k}(\boldsymbol{c}_{1,k})] & if \quad A>Ad \\ \\ Max\{0, Ae^{-\delta}N_1(\boldsymbol{a}_{1,0})-Pe^{-r}N_1(\boldsymbol{b}_{1,0})-ce^{-r}[N_1(\boldsymbol{b}_{1,0})-\tau N_1(\boldsymbol{c}_{1,0})]-c\} & if \quad A\le Ad \end{cases}$$

The firm defaults this time whenever $A_1 \le Ab_1$ , but now the default threshold $Ab_1$ is a non-explicit value. Solving for $Ab_1$ leads to

$$S(A,2)=\begin{cases} G(2,0)-PH(2,0)-\sum_{k=0}^{2} c[H(2,k)-\tau I(2,k)] & if \quad A>Ad \\ \\ Max\left\{0, G(2,0)-PH(2,0)-\sum_{k=0}^{1} c[H(2,k)-\tau I(2,k)]-c\right\} & if \quad A\le Ad \end{cases}$$

$$
= \begin{cases} Ae^{-\delta 2} N_2(\boldsymbol{a}_{2,0}) - Pe^{-r2} N_2(\boldsymbol{b}_{2,0}) - \sum_{k=0}^{2} ce^{-r(2-k)} \left[ N_{2-k}(\boldsymbol{b}_{2,k}) - \tau N_{2-k}(\boldsymbol{c}_{2,k}) \right] & if \quad A > Ad \\ Max\Big\{ 0, Ae^{-\delta 2} N_2(\boldsymbol{a}_{2,0}) - Pe^{-r2} N_2(\boldsymbol{b}_{2,0}) \\ \qquad - \sum_{k=0}^{1} ce^{-r(2-k)} \left[ N_{2-k}(\boldsymbol{b}_{2,k}) - \tau N_{2-k}(\boldsymbol{c}_{2,k}) \right] - c \Big\} & if \quad A \le Ad \end{cases}
$$

In the same way we can find $S(A,t)$, $t = 3, \ldots, T^+$, and $S^-(A,T)$ as $S^+(A,T) - \beta P$.

**Firm:**

We may now compute both $V(A,t)$ and $TB(A,t)$. First note that

$$
V(A,0) = \begin{cases} A & if \quad A > Ab_0 \\ (1-\alpha)A & if \quad A \le Ab_0 \end{cases}
$$

and then

$$
V(A,1) = \begin{cases} G(1,0) + (1-\alpha)\hat{G}(1,0) & if \quad A > Ab_1 \\ (1-\alpha)A & if \quad A \le Ab_1 \end{cases}
$$

$$
= \begin{cases} Ae^{-\delta} N_1(\boldsymbol{a}_{1,0}) + (1-\alpha)Ae^{-\delta}\left[1 - N_1(\boldsymbol{a}_{1,0})\right] & if \quad A > Ab_1 \\ (1-\alpha)A & if \quad A \le Ab_1 \end{cases}
$$

For $t = 2$

$$
V(A,2) = \begin{cases} G(2,0) + (1-\alpha)\sum_{k=0}^{1} \hat{G}(2,k) & if \quad A > Ab_2 \\ (1-\alpha)A & if \quad A \le Ab_2 \end{cases}
$$

$$= \begin{cases} Ae^{-\delta 2} N_2(\boldsymbol{a}_{2,0}) + (1-\alpha) A \sum_{k=0}^{1} e^{-\delta(2-k)} \left[ N_{2-(k+1)}(\boldsymbol{a}_{2,k+1}) - N_{2-k}(\boldsymbol{a}_{2,k}) \right] & if \quad A > Ab_2 \\ (1-\alpha)A & if \quad A \leq Ab_2 \end{cases}$$

and in general, for any $t < T$

$$V(A,t) = \begin{cases} G(t,0) + (1-\alpha) \sum_{k=0}^{t-1} \hat{G}(t,k) & if \quad A > Ab_t \\ (1-\alpha)A & if \quad A \leq Ab_t \end{cases}$$

$$= \begin{cases} Ae^{-\delta t} N_t(\boldsymbol{a}_{t,0}) + (1-\alpha) A \sum_{k=0}^{t-1} e^{-\delta(t-k)} \left[ N_{t-(k+1)}(\boldsymbol{a}_{t,k+1}) - N_{t-k}(\boldsymbol{a}_{t,k}) \right] & if \quad A > Ab_t \\ (1-\alpha)A & if \quad A \leq Ab_t \end{cases}$$

while for $t = T$

$$V(A,T) = G(T,0) + (1-\alpha) \sum_{k=0}^{T-1} \hat{G}(T,k)$$

which leads us to expression (4).

On the other hand

$$TB(A,0) = \begin{cases} \tau c & if \quad A > Ad \\ 0 & if \quad A \leq Ad \end{cases}$$

then

$$TB(A,1) = \begin{cases} \sum_{k=0}^{1} \tau c I(1,k) & if \quad A > Ad \\ \tau c I(1,0) & if \quad Ab_1 < A \leq Ad \\ 0 & if \quad A \leq Ab_1 \end{cases}$$

$$= \begin{cases} \sum_{k=0}^{1} \tau c e^{-r(1-k)} N_{1-k}\left(\boldsymbol{c}_{1,k}\right) & if \quad A > Ad \\ \tau c e^{-r} N_{1}\left(\boldsymbol{c}_{1,0}\right) & if \quad Ab_{1} < A \leq Ad \\ 0 & if \quad A \leq Ab_{1} \end{cases}$$

and in general

$$TB(A,t) = \begin{cases} \sum_{k=0}^{t} \tau c I(t,k) & if \quad A > Ad \\ \sum_{k=0}^{t-1} \tau c I(t,k) & if \quad Ab_{t} < A \leq Ad \\ 0 & if \quad A \leq Ab_{t} \end{cases}$$

$$= \begin{cases} \sum_{k=0}^{t} \tau c e^{-r(t-k)} N_{t-k}\left(\boldsymbol{c}_{t,k}\right) & if \quad A > Ad \\ \sum_{k=0}^{t-1} \tau c e^{-r(t-k)} N_{t-k}\left(\boldsymbol{c}_{t,k}\right) & if \quad Ab_{t} < A \leq Ad \\ 0 & if \quad A \leq Ab_{t} \end{cases}$$

for $t < T$, while $TB(A,T)$ results in expression (5).

Finally, $v(A,t) = V(A,t) + TB(A,t)$ for $t = 0, \ldots, T^{+}$, whereas $v^{-}(A,T)$ is given by (6).

### Debt:

The debt value is the firm value minus the equity value.

For any $t < T$

$$D(A,t)=\begin{cases}(1-\alpha)A\sum_{k=0}^{t-1}e^{-\delta(t-k)}\left[N_{t-(k+1)}(\boldsymbol{a}_{t,k+1})-N_{t-k}(\boldsymbol{a}_{t,k})\right] \\ \qquad +Pe^{-rt}N_t(\boldsymbol{b}_{t,0})+\sum_{k=0}^{t}ce^{-r(t-k)}N_{t-k}(\boldsymbol{b}_{t,k}) & if \quad A > Ab_t \\ (1-\alpha)A & if \quad A \leq Ab_t\end{cases}$$

while $D(A,T)$ is given by (7).■

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