Bond Portfolio Immunization, Immunization Risk and Idiosyncratic Risk

Antonio DÍAZ
María de la O GONZÁLEZ
Eliseo NAVARRO
Departamento de Análisis Económico y Finanzas
Universidad de Castilla-La Mancha
Facultad de CC Económicas y Empresariales de Albacete

Abstract: Classical immunization strategies are based on the hypothesis of parallel term structure changes. So, a different term structure behaviour may affect the effectiveness of immunization. This is known as immunization risk. Previous literature suggests that bullet portfolios allow this risk to be reduced. However, investing in bullet portfolios implies to build up portfolios consisting of only one or two assets and so assuming a high degree of idiosyncratic risk. So, it seems to be some sort of trade off between immunization and idiosyncratic risks. In this paper we develop a set of strategies with different degrees of these two sources of risk and the main outcome is that diversification is the most effective strategy to achieve the objective of immunization, in opposition to previous results where bullet portfolios were considered the optimal strategies against interest rate risk. This result may be a consequence of more realistic hypothesis introduced in the model as well as the fact of using daily prices of actual transactions. The period covered by this study ranges from January 1993 through January 2003 using data from the Spanish Public Debt Market.

Keywords: Investment analysis; Portfolio Immunization; Immunization risk; Idiosyncratic risk.

JEL Classification: C61, E43, G11, G12

Resumen: Las estrategias de inmunización clásica están basadas en la hipótesis de variaciones paralelas de la estructura temporal de los tipos de interés. Así, comportamientos de la estructura temporal distintas a la supuesta podrían afectar a la eficacia de la inmunización. Esto es conocido como riesgo de inmunización. La literatura previa sugiere que las carteras bullet permiten reducir este riesgo. Sin embargo, invertir en este tipo de carteras implica concentrar la inversión en tan solo uno o dos títulos y de este modo, asumir un alto grado de riesgo idiosincrásico. En este trabajo hemos desarrollado un conjunto de estrategias con diferentes niveles de exposición a estas dos fuentes de riesgo y la principal conclusión es que la diversificación es más importante que la reducción del riesgo de inmunización para conseguir el objetivo de inmunización, en contraposición a los resultados previos donde las carteras bullet se consideraron las estrategias óptimas frente al riesgo de tipo de interés. Este resultado puede ser una consecuencia de las hipótesis más realistas incluidas en el modelo así como el hecho de utilizar precios diarios de transacciones reales. El

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periodo cubierto por este estudio está comprendido entre enero de 1993 y enero de 2003, utilizando datos del Mercado Español de Deuda Pública.

Título: Inmunización de Carteras de Bonos, Riesgo de Inmunización y Riesgo Idiosincrásico

Palabras clave: Análisis de inversión; Inmunización de carteras; Riesgo de inmunización; Riesgo idiosincrático.

JEL Classification: C61, E43, G11, G12

1. INTRODUCTION

It is well known that the objective of an immunization strategy is to guarantee a final portfolio value independently of interest rate changes. However, one of the main drawbacks of these strategies against interest rate risk is the fact that they are based on a particular hypothesis about the behaviour of the yield curve. For instance, in the seminal paper of Fisher and Weil (1971) the immunization strategy was designed to protect just against parallel shifts of the term structure. Other authors developed alternative duration formula,[1] each one derived from a different assumption about the behaviour of interest rates. [2]

Thus, if interest rates shift in a way different from that assumed, immunization may fail to guarantee the promised return at the end of the investor holding period which is the objective these strategies were intended for.[3]

This risk is known as immunization risk and, since the early eighties, some authors have tried to work out how to minimise the impact of these unexpected term structure changes. Fong and Vasicek (1983, 1984) obtained one of the first results in this area. They proposed an immunization risk measure, M-Square, that can be understood as a variable that tries to account for the dispersion of a portfolio cash flows around the end of investor planning period. According to these authors, the smaller the portfolio M-Square, the lesser the immunization risk. As a trivial case, a portfolio consisting of zero coupon bonds maturing at the end of the investor holding period is an immunised portfolio free of immunization risk and with a M-Square equal to zero.

After this pioneer work, other studies led to different measures of immunization risk, the differences being caused by alternative assumptions about what can be considered as an adverse

1 See Bierwag (1987) for a general review of these strategies.

2 A popular approach is to assume that interest rate changes can be accurately described by shifts in a limited number of risk factors. These models correspond to the partial duration models (Reitano, 1990), the common factor duration models (Litterman and Scheinkman, 1991), the optimal key-rate duration model (Ho, 1992), among others. An alternative approach uses parametric duration models from zero-coupon rate functions. Cooper (1977) first assumes that each change in the interest rate corresponds to a change in one of the parameters that make up the interest rate term structure. For instance, Garbade (1985) and Chambers et al. (1988) assume a polynomial fitting of the term structure and Willner (1996) uses the Nelson and Siegel (1987) exponential model. Also, non-arbitrage term structure models as Vasicek (1977) or Cox et al. (1979) can be used to derive duration formula.

3 The promised return is the spot rate with a term equal to the investor's horizon.

term structure twist.[4] In any case, all these strategies suggest building up bullet portfolios that concentrate their cash flows around the end of the investor planning period. In the extreme case, following these investment strategies would imply to buy a single bond with maturity as close as possible to the end of the investor horizon. Moreover, Nawalkha and Chambers (1996) suggest that the best strategy to minimise both interest rate risk and immunization risk is to concentrate the investment on the bond with the closest maturity at the end of the planning period even though its duration would be smaller than the investor horizon.

However, this policy would imply to invest in a non-diversified portfolio as far as in most markets it is not possible to find more than two or three liquid enough bonds with the required characteristics. This would mean to assume a high level of idiosyncratic risk that in the bond markets may arise from different sources as the interaction between bonds and derivative contracts (as cheapest to delivery effects), coupon washing effects, strips operations, status changes, etc. We explain widely this key point in the next paragraphs.

Actual bond prices can be obtained as the sum of two components: the present value of the expected cash flows plus an error term that accounts for idiosyncratic elements which are independent of interest rates. Among these idiosyncratic factors that can affect bond prices, we have tax effects derived from coupon washing, strips operations where some bonds can be involved, status changes from on-the-run to off-the-run which may affect bond liquidity, changes in the cheapest to deliver bond in future contracts, etc. All these factors affect bond prices regardless of interest rate changes and so may affect immunization strategies.

In a well diversified bond portfolio, these error terms should tend to cancel out as far as they are independent of interest rate changes and they are independent of each other. In any case, the impact on the portfolio of a deviation from the theoretical value caused by a particular bond would be smaller if that bond has a portfolio weight of say 15% than if the weight is over 50%. This could be the case of a bullet portfolio consisting of only one or two bonds. Moreover, the variance of the mean of the error terms should decrease as we increase the number of bonds included in the portfolio.

Concerning immunization strategies, these idiosyncratic effects would be more intense in non diversified portfolios. If we measure the effectiveness of immunization strategies by the standard error of portfolio returns over the investor horizon, there is a clear bias favourable to diversified portfolios. According to previous literature, term structure twist would affect in a bigger degree ladder portfolios. But if this ladder portfolio is composed by several bonds then, the idiosyncratic factors would have a smaller effect. If these idiosyncratic effects are bigger than the effect of term structure twists, diversified portfolios would tend to present more stable returns.

To illustrate the effect of diversification on portfolio value, we represent in the next figure the difference between actual bond prices and their theoretical price on the basis of discounted cash flows using a previously estimated term structure of interest rates.

4 See, for instance, Nawalkha and Chambers (1996); Nawalkha and Chambers (1997); Nawalkha et al. (2003); Balbás and Ibáñez (1998) or Balbás et al. (2002).

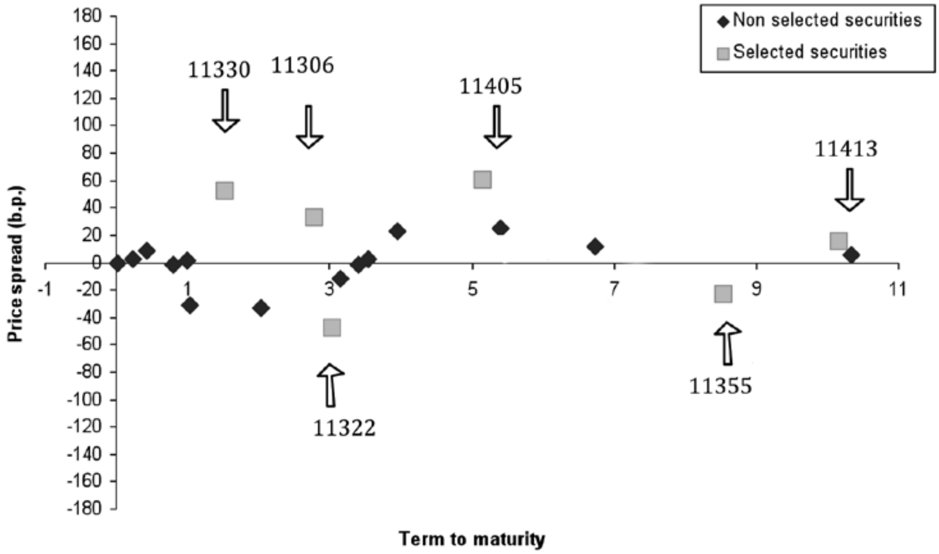
A bullet strategy would select bonds 11306 and 11405. As we can see, both bonds seem to be overvalued. However, a more diversified strategy would choose all bonds marked in red. So, a well diversified portfolio would contain overvalued and undervalued bonds and pricing errors would tend to compensate each other. In the end, that could imply a less volatile portfolio return, as the results of the paper suggest.

Table 1. Initial optimal portfolios for alternative strategies for the third three-year period (July 93-July 96)

		Bond characteristics					
Issue code		11330	11306	11322	11405	11355	11413
Maturity date		95.01.16	96.04.15	96.07.15	98.08.30	02.01.15	03.08.30
Coupon (%)		11.8	13.45	11.9	11.45	11.3	10.9
		Bond Weights (%)					
Strategy	Portfolio duration	ω1	ω2	ω3	ω4	ω5	ω6
Model 1 (λ=1)	3	0	0.6967	0	0.3033	0	0
Model 3 (λ=0.9)	3	0	0.4440	0.2529	0.3031	0	0
Model 3 (λ=0.8)	3	0	0.3910	0.3060	0.3030	0	0
Model 3 (λ=0.7)	3	0	0.3733	0.3237	0.3030	0	0
Model 3 (λ=0.6)	3	0	0.3644	0.3325	0.3030	0	0
Model 3 (λ=0.5)	3	0	0.3612	0.3399	0.2939	0.0050	0
Model 3 (λ=0.4)	3	0	0.3648	0.3505	0.2620	0.0227	0
Model 3 (λ=0.3)	3	0.0361	0.3443	0.3351	0.2362	0.0484	0
Model 3 (λ=0.2)	3	0.1263	0.2922	0.2868	0.2050	0.0759	0.0138
Model 3 (λ=0.1)	3	0.2005	0.2522	0.2498	0.1749	0.0864	0.0362
Model 2 (λ=0)	3	0.2599	0.2203	0.2202	0.1508	0.0948	0.0541
Model 4 (Min MA)	2.7	0	0	1	0	0	0

Figure 1

Difference between actual bond prices and theoretical prices



So, there is a trade off between immunization risk and idiosyncratic risk. The question that immediately arises is which of these two risks is more relevant with respect to the main purpose of an investor who is planning to undertake an immunization strategy (to guarantee a final portfolio value).

In order to test the relative importance of these two sources of risk, different issues should be considered.

First, it should be pointed out that most studies use bond prices of "synthetic" assets created from term structure estimations to check the effectiveness of immunization strategies. Just few studies use actual bond prices to test immunization strategies. In Fooladi and Roberts (1992) quoted prices as well as prices obtained from fitted term structures are used to test alternative immunization strategies for the Canadian default free bond market (1963-1986). Soto (2004) uses bond prices of actual transactions of the Spanish Public Debt Market. This database is similar to our own but there exist a number of differences between the use of both databases. Soto considers a shorter sample period (1992-1999) and a brief investor horizon (12-24 months). Also Soto's analysis just includes long term bonds, allowing short positions in order to obtain immunized portfolios. Because of that, the studies that use these prices of "synthetic" assets eliminate the idiosyncratic risk inherent in all the individual bonds or portfolios with a reduced number of assets. So, the potential problems derived from investing in non-diversified portfolios can just be tested if the data used consist of bond prices of actual transactions. So, in this study we use average daily prices of all actual transactions, that is, we use actual bonds instead of "synthetic" bonds.[5]

5 Fooladi and Roberts (1992), for instance, used actual prices averaged on a monthly basis. It would have been preferably to use prices of actual single transactions to consider the real idiosyncratic risk but these data were not available in our database. The fact of using daily averages probably misestimates the actual idiosyncratic risk.

A second issue that may influence the results is the portfolio design. Most tests proceed to rebalance portfolio holdings periodically (usually, every three or six months). In this paper, however, portfolio is restructured whenever any payment (coupon or principal) of any of the six available bonds (the opportunity set of bonds) is due, adding more realistic features to the investment process. Also, it should be highlighted that data from the repo market are eventually used for implementing immunization in the final stages of the investor horizon.

Finally, most studies introduce short sales in order to overcome the difficulties of adjusting portfolio duration to the bonds available. On the contrary, we assume more realistic hypothesis and we have not allowed short sales as there are important institutional constrains to apply them in practice. Then, we adjust portfolio duration during the investor horizon.

To test this point we propose an optimization program that selects among immunized portfolios (those portfolios with a duration equal to the investor horizon) those with the smallest M-Absolute, the dispersion measure developed by Nawalkha and Chambers (1996). Alternatively, we apply a second model that chooses the immunized portfolio that generates cash-flows with the greatest dispersion. Then, a mixed model is developed through a program which allows weighting the relative importance of idiosyncratic and immunization risk in selecting immunized portfolios. As a benchmark, we also present the strategy developed by Nawalkha and Chambers (minimum M-Absolute without matching portfolio duration).

All these immunization models will be applied to Spanish government bond market data. This market is interesting for several reasons. First, it is the forth largest market within the Euro area with an outstanding debt volume close to €315 billion at the end of the sample period (January 2003). Nowadays, the sovereign Spanish debt has the highest quality according to all rating agencies with an outstanding volume around the 34 % of the Spanish GDP. All these arguments make Spanish Public Debt assets a very close substitute of the highest quality euro area paper and particularly of the German Treasury assets. But what makes Spanish market specially interesting for this analysis are the dramatic changes in interest rates (in both level and shapes of the term structure) that market experienced during the sample period (from 1993 to 2003) due to the convergence process of the Spanish economy to the new European monetary area. So, we believe that the immunization risk born by the Spanish term structure during this period was extremely high. To check this point we carried out a factorial analysis of the term structure that indicates that during the sample period parallel shifts only accounted for around a 78 % of the variance of monthly interest rate changes, a percentage well below other markets have gone through.[6]

So, this empirical study is, a priori, somehow favourable to yield better results to those models which just take into account immunization risk for two reasons. First, because idiosyncratic risk has been partly removed when using mean daily actual transaction prices instead of single transactions prices and, second, because the period covered by this study, especially the first half, was extremely volatile in both level and shapes of the term structure. However, we show that there is evidence of the higher success of the diversification with respect to the concentration to achieve the objective of immunizations strategies.

6 See for instance Litterman and Sheinkman (1991) who found for the US Treasury market that the first factor (parallel shifts) account on average for a 89.5 % of the variance of interest rates changes.

Then in this paper we proceed as follows. First we describe the set of data used and proceed to analyse the behaviour of the term structure during the sample period applying factorial analysis techniques. Then we describe the procedures employed to design bond portfolio selection process and present the results obtained for the alternative strategies considered in this paper. Finally, we derive the main conclusions.

2. DATA

The data used in this study consist of bonds, bills and repo mean daily prices of actual transactions from the Spanish Public Debt Market over the period from January 1993 through January 2003. These data are provided by Banco de España.[7] Daily transactions involve between 29 to 33 Treasury bonds comprising more than 66 different assets during the whole sample period. Bills are also included in this study as well as one-week repo market operations.

To have a hint of the behaviour of interest rates in the Spanish market during this period we have proceed to estimate everyday the term structure of interest rates.[8] The summary statistics of monthly changes of interest rates are given in Table 2. The level of one-month, one-year and ten-year interest rates are represented in Figure 2. It can be clearly seen the dramatic decrease of interest rates and the twist of the yield curve during the sample period and particularly during the first half. As expected, short rates have a greater volatility than long term interest rates.

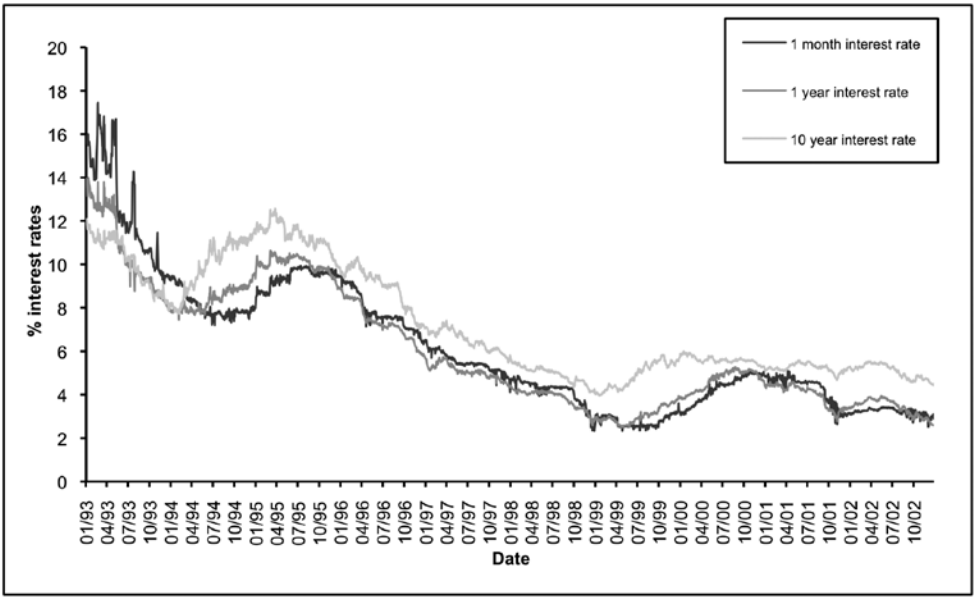
Table 2. Summary statistics of monthly changes of interest rates from term structure estimations in the Spanish Public Debt Market

	Jan 93-Jan 03			Jan 93-Dec 98			Jan 99- Jan 03		
	1month	1 year	10 year	1month	1 year	10 year	1month	1 year	10 year
Mean (%)	-0.099	-0.089	-0.058	-0.174	-0.147	-0.102	0.012	-0.006	0.005
Stand.Dev.	0.630	0.330	0.314	0.787	0.379	0.373	0.232	0.219	0.185
Median	-0.030	-0.120	-0.050	-0.173	-0.143	-0.076	0.033	0.00002	-0.012
Maximum	2.069	0.581	0.755	2.069	0.581	0.755	0.370	0.466	0.391
Minimum	-4.049	-2.287	-1.142	-4.049	-2.287	-1.142	-0.591	-0.545	-0.372

7 See www.bde.es/banota/series.htm

8 Daily estimates were made using Nelson and Siegel (1987) methodology applied to mean daily prices of all bond and bills traded each day.

Figure 2. Level of one-month, one-year and ten-year interest rates



Also, we have carried out a factorial analysis of the yield curve.[9] The first three factors can be identified (as usual) as parallel, slope and curvature changes of the yield curve and they account for a 77.94, 15.69 and 4.68 % of the total variance respectively. So parallel shifts seem to explain in a lesser degree the behaviour of the term structure compared with other countries and so it seems that immunization risk was greater than in other default free bond markets.[10]

In order to test the effectiveness of different strategies against interest rate risk we specify a planned holding period of three years dividing the sample period into twenty nine overlapping intervals each one starting the first trading day of January, April, July and October from January 1993 to January 2000.

3. PORTFOLIO DESIGN

Initially, the opportunity set within which bonds are chosen to build up immunized portfolios consists of six Treasury bonds with the highest liquidity:[11] two bonds with maturity before investor horizon, a three year bond, a ten year bond and two more bonds with maturity longer than three years.

9 Monthly changes in one, three, six and twelve month interest rates and two year to ten year interest rates were used as inputs to undertake factorial analysis.

10 Recall that immunization risk is the risk of not obtaining the target return (or promised return) at the end of the investor holding period when carrying an immunization strategy due to term structure movements different from those assumed.

11 Liquidity is measured by its trading volume. For a more detailed analysis of the liquidity of the Spanish Treasury bond market and other related institutional issues see, for instance, Díaz et al. (2006).

One of the achievements of this paper with respect to previous studies is that, once we have selected the initial portfolio, it is rebalanced each time any of the six available bonds (the ones which constitute the opportunity set of bonds) pays a coupon or matures, reinvesting these payments among the chosen bonds to keep portfolio immunized.[12] When bonds mature, the program is run again replacing the old bonds by others with maturity before the end of the holding period.[13] Another problem we have to deal with in order to implement immunization is that when approaching the end of the holding and the bonds held in the portfolio are close to maturity its liquidity decreases dramatically.[14] So, when we are next to the end of the holding period we can not find any transaction for bonds with the suitable term to maturity. In this case, we consider bills. Anyway, sometimes neither bills nor bonds are often found for very short maturities. In these cases, we reinvest the money in one-week repo operations. This involves that portfolio must be rebalanced a minimum of once a week until the end of the holding period. Anyway, it should be pointed out that prices of bonds, bills and repos correspond to the daily mean of actual transactions.

4. METHODOLOGY

As the aim of this paper is to compare the impact of immunization and idiosyncratic risk on immunized portfolios, we have developed initially two different programs.

The first one selects, among the opportunity set described in the former section, the portfolio with duration equal to the investor holding period that minimises a measure of immunization risk.[15] In this case, this measure is the M-Absolute or M^A. Nawalkha and Chambers (1996) developed this measure. According to these authors portfolio M-Absolute is defined as:

$$M^A = \frac{\sum_{i=1}^{m} |t_i - H| \cdot C_i \cdot P_0(t_i)}{I_0} \tag{1}$$

Where C_i is the cash flow due at t_i generated by the portfolio $(C_i > 0)$, $P_0(t_i)$ is the present value of a unit zero coupon bond with maturity *at* t_i, I_0 is the portfolio present value and H is the length of the horizon. So M^A is a measure of the deviations of portfolio cash flows around the end of the investor holding period. It can be easily shown that a portfolio M-Absolute is the weighted average of the M^A of each individual asset.

12 Usually, the most common assumption when testing immunization strategies is that portfolio weights are adjusted periodically. For instance, Fooladi and Roberts (1992) assume semiannual rebalancings meanwhile Soto (2001, 2004) assumes quarterly adjustments. This procedure may have an important impact especially if interest rates fluctuate sharply as it happened in the Spanish market during the period 1993-1998.

13 We determine, at the beginning of each subperiod, all the dates where any coupon of the six initial bonds is paid or any asset matures (they can be bonds, bills or repo). Then, we rebalance the portfolios in all these dates, regardless of the strategy followed or the composition of the portfolio.

14 See, for instance, Díaz et al. (2006) or Sarig and Warga (1989).

15 We use the Fisher and Weil (1971) duration formula. However, we have also used duration formula derived from Vasicek (1977) and Cox et al. (1979), but these models led to similar results than the simpler and easier to compute Fisher and Weil duration formula.

Then, this strategy, aimed to minimise immunization risk, is modelled through the following linear program[16]:

Model 1

$$\text{Min} \quad \sum_{j=1}^{N} \omega_j \cdot M_j^A \qquad (2)$$

$$\sum_{j=1}^{N} \omega_j \cdot D_j = H$$

$$\text{subject to} \quad \sum_{j=1}^{N} \omega_j = 1$$

$$0 \leq \omega_j \leq 1, \forall j$$

where N is the number of bonds available, M_j^A and D_j are the M-Absolute and the Fisher & Weil's duration of bond j respectively,[17] ω_j is the weight of bond j (short sales are not allowed) and H is the investor horizon.

The second program has as objective function the sum of the bond squared weights spreading out the investment among the available assets and then minimising the effects of unsystematic risk.[18]

Thus, this second program becomes:

Model 2

$$\text{Min} \quad \sum_{j=1}^{N} \omega_j^2 \qquad (3)$$

$$\text{subject to} \quad \sum_{j=1}^{N} \omega_j \cdot D_j = H$$

$$\sum_{j=1}^{N} \omega_j = 1$$

$$0 \leq \omega_j \leq 1, \forall j$$

Then, we consider a mixed strategy to test if there is a trade off between immunization and idiosyncratic risks. This new program consists of changing the objective function by a convex combination of the two former objective functions.

16 Bertocchi et al. (2005) proposed another linear programming problem to immunize bond portfolios from changes of the yields of corporate bonds.

17 Fisher and Weil's portfolio duration is defined as $\sum_{i=1}^{N} t_i \cdot C_i \cdot P_0(t_i) / I_0$ where t_i, C_i, $P_0(t_i)$ and I_0 are defined as before.

18 This objective function has been already used in Chambers et al. (1988).

Then this mixed strategy is model through the following program:

Model 3

Min $$\text{Min } \lambda \sum_{j=1}^{N} \omega_j \cdot M_j^A + (1-\lambda) \cdot \sum_{j=1}^{N} \omega_j^2 \qquad (4)$$

subject to $$\sum_{j=1}^{N} \omega_j \cdot D_j = H$$

$$0 \le \omega_j \le 1, \forall j$$

where λ is a parameter between zero and one. For lambda equal to one the objective function is the same than that of the first model and the program solution is the immunized portfolio with minimum M-Absolute. If lambda takes value equal to zero we get the second model and then the program provides the immunized portfolio with the maximum diversification. Other values of lambda represent intermediate positions between the two extreme cases.

Finally, as a benchmark, we also perform the strategy, that according to Nawalkha and Chambers (1996) best, allows to reduce simultaneously interest rate risk and immunization risk: to minimise portfolio M-Absolute without taking into account its duration.

This strategy is modelled as follows:

Model 4

Min $$\text{Min } \sum_{j=1}^{N} \omega_j \cdot M_j^A \qquad (5)$$

subject to $$\sum_{j=1}^{N} \omega_j = 1$$

$$0 \le \omega_j \le 1, \forall j$$

Usually, this strategy leads to select a portfolio consisting of a single bond with maturity as close as possible to the end of the investor planning period.

5. RESULTS

To illustrate the difference between the former models, we show in Table 3 the weights of the optimal portfolios selected when applying the four models to the first three-year investor holding period (January 1993 to January 1996). For the mixed strategy (model 3) we present the initial portfolio for different values of parameter lambda (recall that for $\lambda = 1$ model 3 nests model 1 and for $\lambda = 0$ model 3 nests model 2).

Table 3. Initial optimal portfolios for alternative strategies for first three year period (January 93-January 96)

Bond characteristics							
Maturity date		95.01.16	95.07.17	96.04.15	96.07.15	97.01.15	02.06.15
Coupon (%)		11.8	11.4	13.45	11.9	11.6	10.3
Bond Weights (%)							
Strategy	Portfolio duration	ω1	ω2	ω3	ω4	ω5	ω6
Model 1 (λ=1)	3	0	0	0	0.9732	0	0.0268
Model 3 (λ=0.9)	3	0	0	0.0930	0.8709	0	0.0360
Model 3 (λ=0.8)	3	0	0	0.3064	0.6319	0.0047	0.0570
Model 3 (λ=0.7)	3	0	0	0.3203	0.4992	0.1280	0.0525
Model 3 (λ=0.6)	3	0	0.0650	0.2980	0.4073	0.1660	0.0637
Model 3 (λ=0.5)	3	0	0.1251	0.2750	0.3439	0.1811	0.0748
Model 3 (λ=0.4)	3	0	0.1652	0.2597	0.3017	0.1912	0.0822
Model 3 (λ=0.3)	3	0.0144	0.1878	0.2439	0.2674	0.1948	0.0916
Model 3 (λ=0.2)	3	0.0818	0.1815	0.2130	0.2259	0.1831	0.1147
Model 3 (λ=0.1)	3	0.1341	0.1765	0.1890	0.1936	0.1740	0.1327
Model 2 (λ=0)	3	0.1760	0.1756	0.1698	0.1677	0.1667	0.1472
Model 4 (Min MA)	2.62	0	0	1	0	0	0

First, we indicate the main features of the six bonds traded in the Spanish Treasury market with the required characteristics: two bonds with duration smaller than the investor horizon, a three year bond, two bonds with duration longer than the holding period and a ten year bond. The selected bonds are the most liquid ones among those within each category.

It can be seen that the smaller the value of the parameter λ the bigger the number of bonds included in the initial portfolio. For instance, for λ=1, there are two assets included in the portfolio. For λ=0.9 there would be three bonds, for λ=0.8 four, for λ=0.6 five and for λ less than 0.3, the number of bonds in the initial portfolio would be six.

Then, strategy 1 selects the immunised portfolio with the minimum dispersion meanwhile strategy 2 spreads out the initial investment among the six bonds considered. Strategy 3 provides intermediate solutions between both extremes and, finally, strategy 4 concentrates all initial investment in the bond with the smallest M-Absolute but with duration smaller than the investor horizon.

As stated before, this program is applied each time any cash flow is generated and when bonds mature they are replaced by others with maturity before the end of the remaining holding period or, if it was not possible, by bills or, eventually, by one-week “repo” operations.

To compare the effectiveness of these strategies we calculate the difference between the realized return and the target return (three year zero coupon bond yield at the beginning of the holding period) when following each strategy. As the objective of immunization is to guarantee the target return we consider that, for a given period, a strategy outperforms an alternative one if this deviation, in absolute terms, is smaller. For the whole sample period (that is for the 29 holding periods) we also calculate the standard deviation of these differences. A smaller standard deviation would indicate a greater effectiveness in accomplishing the investor objective. So, we are following a maximin criterion[19].

Table 4 provides the summary statistics of the target return (three year zero coupon yield) and its three year changes over the 29 holding periods. Table 4 shows that changes in the three year interest rates have been dramatic over the sample period with a striking maximum (in absolute terms) change of 736 basis points during the period April 1995-April 1998.

Table 4. Summary statistics of target returns (three-year zero coupon bond yields)

	Level (%)			Three year changes		
	Jan93-Jan98	Jan98-Jan03	Jan93-Jan03	Jan93-Jan98	Jan98-Jan03	Jan93-Jan03
Mean	8.87	4.18	6.47	-3.60	0.07	-2.46
St. Dev.	2.33	0.67	2.91	2.42	0.99	2.69
Median	8.81	4.35	5.13	-3.49	0.16	-1.77
Maximum	13.05	5.25	13.05	0.30	1.53	1.53
Mínimum	4.95	3.07	3.07	-7.36	-1.77	-7.36

As we can see, the volatility of the target return has been much higher during the first six years that correspond approximately to the convergence period of the Spanish currency to the Euro. So, we have split up the sample period into two subperiods to check if the different behaviour of interest rates during these two periods has influenced the effectiveness of the different strategies. [20]

19 Balbás e Ibáñez (1998) and Balbás, Ibáñez and López (2002) also work with immunization strategies built around the maximin criterion and Ghezzi (1999) proposes a maximin optimal control problem applied to an example of immunization strategies.

20 Although the new European currency came officially into effect the first of January 1999, the convergence process towards the new monetary area was established before that date. The convergence of forward rates between Spanish and central European economies was accomplished at least one year before. So we find more adequate to subdivide the sample period in January 1998.

Table 5 report mean and standard deviations of the absolute difference between realized and target returns.[21] Then, according to these results and our maximin criterion, the best strategy would be strategy 2 which consists of diversifying as much as possible the investment. This strategy can be considered as a ladder portfolio. Then, the optimal value of lambda is λ = 0. This outcome is valid not only for the whole sample but for both subperiods considered in this study. It can be seen that the mixed strategy (model 3) yields intermediate results between strategies 1 and 2.

Table 5. Realized vs. Target Returns by Strategies. Three year holding periods January 93-January 03

Strategy	Mean absolute difference (%)			St. dev. of absolute difference (%)			Number of periods outperforms model 4		
	93.01-98.01	98.01-03.01	93.01-03.01	93.01-98.01	98.01-03.01	93.01-03.01	93.01-98.01 20 periods in total	98.01-03.01 9 periods in total	93.01-03.01 29 periods in total
Model 1 (λ=1)	1.24	0.66	1.06	1.17	0.57	1.05	12	7	19
Model 3 (λ=0.8)	1.00	0.64	0.88	0.99	0.55	0.88	16	6	22
Model 3 (λ=0.6)	0.79	0.51	0.70	0.75	0.33	0.65	18	6	24
Model 3 (λ=0.4)	0.63	0.41	0.56	0.55	0.30	0.49	17	6	23
Model 3 (λ=0.2)	0.54	0.36	0.48	0.43	0.27	0.39	16	6	22
Model 2 (λ=0)	0.44	0.32	0.40	0.31	0.22	0.29	17	6	23
Model 4 (Min MA)	1.19	1.08	1.16	1.06	0.82	0.98	-	-	-

On the contrary, the worst results for the whole sample period correspond to models 1 and 4.[22] Then, diversification seems to help to reduce risk more than the strategies followed to prevent term structure twists. This is an important finding as far as this result seems to contradict previous studies where bullet portfolios were used to provide the most efficient results.[23]

In order to test if the outcomes of Table 5 depend on the behaviour of interest rates (particularly the size and sign of interest rates shifts) we estimate the following regressions for each strategy where we analyse if the difference between realized and target returns depends on the changes of the target return.

$$R_t^{realized} - R_t^{target} = \beta_0 + \beta_1 \cdot \Delta R_t^{target} + u_t \qquad t=1, 2, \ldots, 29 \qquad (6)$$

21 Similar results are obtained if squared differences between target and actual returns are used instead of the absolute value of these differences.

22 Moreover, if actual deviations (instead of its absolute values) are used to analyse the effectiveness of those strategies, model 4 clearly obtains the worst results.

23 In Fooladi and Roberts (1992) bullet immunized portfolios outperform significantly ladder portfolios for the Canadian market and Soto (2004) find similar results for the Spanish default free bond market.

where $R_t^{realized}$ is the realized portfolio return for each period, R_t^{target} is the target return (three year zero coupon yield) at the beginning of each three year period (investor horizon), ΔR_t^{target} is the change in the target return during the investor horizon period and u_t is an error term.

We can not reject the null hypothesis H_0: $\beta_1 = 0$ in any of these strategies because p-values > 0.05 in all the cases.

Another interesting point in this study is that, so far, we have not allowed short sales. Most studies introduce short sales in order to overcome the difficulties of adjusting portfolio duration with the bonds available. So we rerun strategies 2 and 3 (for λ = 0.2, 0.4, 0.6, 0.8) to check if this fact has any influence on the relative performance of these strategies. Results are shown in Table 6. As can be seen comparing Table 5 and Table 6, allowing short sales does not seem to have any relevant impact on risk reduction, which is an outcome already reported in Fooladi and Roberts (1992).

Table 6. Realized vs. Target Returns by Strategies with short sales allowed. Three year holding periods January 93-January 03

Strategy	Mean absolute difference (%)			St. dev. of absolute difference (%)		
	93.01-98.01	98.01-03.01	93.01-03.01	93.01-98.01	98.01-03.01	93.01-03.01
Model 3 (λ=0.8)	1.85	1.30	1.68	1.50	0.84	1.34
Model 3 (λ=0.6)	0.97	0.74	0.90	0.86	0.48	0.76
Model 3 (λ=0.4)	0.65	0.51	0.60	0.57	0.36	0.51
Model 3 (λ=0.2)	0.51	0.40	0.47	0.41	0.28	0.37
Model 2 (λ=0)	0.44	0.34	0.41	0.32	0.22	0.29

6. SUMMARY AND MAIN CONCLUSIONS

When implementing immunization strategies investor has to deal with the risk of term structure movements that can make impossible to achieve the objective these strategies were intended for. This objective is to guarantee a final portfolio value independently of interest rate changes. Many authors suggested to develop several techniques in order to minimise the effects of adverse term structure changes. These techniques usually imply to concentrate investment in one or two bonds, leading to portfolios with a high degree of idiosyncratic risk. So, we have checked which of these two sources of risk, immunization risk and idiosyncratic risk, has a greater impact on bond returns or more precisely on the deviations of realized bond portfolio returns with respect to the target return. So we have tested four different strategies using linear programming techniques which are shown to be an extremely useful instrument in bond portfolio management. Strategy 1 which leads to immunized portfolios with maximum concentration around the investor horizon, strategy 2 that also selects immunized portfolios but with maximum diversification, strategy 3 which can be considered as a mixture of the two former strategies and finally, strategy 4 as benchmark we test the Nawalkha and Chambers' immunization strategy. These authors suggest to reduce simultaneously interest risk and immunization risk by minimising a dispersion measure, M-Absolute.

The main contribution of this paper is that we have introduced more realistic features to the portfolio design and its rebalancing process. First, we have used prices of actual transactions, allowing to take into consideration idiosyncratic risk which may disappear if we had used fitted term structure bond prices. Second, we proceed to rebalance portfolio weights each time a cash flow is due instead of assuming periodical portfolio adjustments. The assumptions made about cash flow reinvestments within each rebalancing period may have important effects on the results. Third, short sales are not allowed. To overcome the problems this fact may cause when adjusting portfolio duration during the investor horizon, especially when shorter bonds mature, we let bills and repo operations to enter portfolio. This is a more realistic feature than keeping short positions in bonds (when allowed) for long periods of time.

The main result is that diversification helps, in a greater degree than concentration, in reducing the volatility of the deviations between target and realized returns. Also this result is independent of the size and sign of interest rate changes. Finally, we find that immunized portfolios outperform those strategies based on minimising M-Absolute.

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