A factor analysis of volatility across the term structure: the Spanish case

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Abstract: A major problem for risk management in fixed income portfolios is the large dimensionality of the implied vector interest rate. This is specialy important for VAR computation, for which an estimate of the variance-covariance matrix of interest rates needed. Alexander (2000) suggested computing that matrix from a factor model for interest rates, sharply reducing the dimensionality of the problem. We start by showing that the term structure of volatilities for zero-coupon interest rates from the Spanish secondary debt market can also be explained by a reduced number of factors. This factor representation can be used to produce volatility time series across the whole term structure that are shown to significantly differ from those obtained under Alexander's approach. Furthermore, the latter is shown to underperform our alternative proposal, which is similar to the ad-hoc estimation rules in Riskmetrics. Reducing the dimensionality of the vector interest rates seems to lead to a relevant loss of information regarding second order moments that does not occur in alternative approaches that use the information in the full vector.

Key words: interest rate, factor model, volatility and Value at Risk.

JEL Classification: E43, G11.

Resumen: La elevada dimensión de la Estructura Temporal de Tipos de Interés(ETTI) es uno de los principales problemas asociados a la gestión del riesgo en carteras de renta fija. Este problema es especialmente importante de cara al cálculo del Valor en Riesgo (VaR), ya que para ello se requiere calcular la matriz de varianzas y covarianzas del conjunto de tipos de interés que determinan el valor de la cartera. Alexander(2000) propone una forma sencilla de calcular dicha matriz mediante un modelo de factores de la ETTI. En este trabajo nosotros comenzamos mostrando que la estructura temporal de volatilidades del mercado español de deuda pública puede ser explicada a partir de un número reducido de factores. Esos factores pueden ser utilizados para reproducir las series temporales de

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volatilidad de todos los tipos en la estructura temporal y mostramos que esas estimaciones son estadísticamente distintas a las obtenidas mediante la aproximación propuesta por Alexander (2000). Adicionalmente, mostramos que en términos de VaR la propuesta de Alexander (2000) funciona algo peor que la nuestra que ofrece resultados similares a los obtenidos por Rismetrics. Los resultados obtenidos sugieren que reducir la dimensión de la ETTI puede llevar a una pérdida importante de información de cara a estimar los segundos momentos condicionales de los tipos de interés.

Palabras clave: tipos de interés, estructura temporal, modelo de factores, volatilidad, Valor en Riesgo.

Código JEL: E43, G11.

1. INTRODUCTION

Searching for a sensible factor representation of the term structure of interest rates has been object of study for some time. If interest rates at any given maturity could be written, to a reasonable approximation, as a linear combination of a small number of factors, then fluctuations of the yield curve could be characterized by just analyzing the behaviour of the chosen factors. These could either be rates of return for specific maturities, like the one month rate, simple linear combinations of them, like the spread between a long- and a short-term rate, or more complicated linear combinations of interest rates at different maturities. In particular, interest rate forecasts for every maturity could be derived from forecasts for the factors.

With some differences across a variety of international fixed income markets, this type of analysis concludes in a positive note, by characterizing a small number of factors able to represent, to a large extent, the behaviour of the term structure of interest rates (Stock and Watson [1988], Elton, Gruber and Michaely[1990], Litterman and Scheinkman [1991], Hall, Anderson and Granger [1992], Zhang [1993], Kahn y Gulrajani [1993], D'Ecclesia and Zenios [1994], Engsted and Tanggaard [1994], Navarro y Nave [1995], Barber and Copper [1996], Bliss [1997], Navarro and Nave [1997], Domínguez and Novales[2000]).

This line of research was originally proposed to reduce the dimensionality of a usually large vector of interest rates by obtaining a simple linear representation of the term structure. However, there is some sense in which representing interest rates levels by a small number of factors also leads to a simple representation of interest rates fluctuations. This is why sometimes a reference is made to the fact that the factor representation is a representation of interest rates as well as a representation of volatilities across the term structure. Alexander (2000) exploited this possibility by suggesting the use of a factor interest rate model to compute their conditional variance-covariance matrix as explained below.

On the other hand, if we have a set of time series for estimated volatilities for each of a large set of maturities across the term structure, we can directly search for a factor representation of the set of volatility time series. We start by showing that, as it is the case with interest rates themselves, there is a clear possibility of reducing the information contained in the vector of their conditional volatilities to a small number of factors. But then, maybe contrary to a simple intuition, we show that the volatility series estimated from a factor model for interest rates differ significantly from those obtained from a factor model for volatilities. This observation is clearly very relevant for many issues related to risk management in fixed income markets. Taking VAR computation as an illustration, we show that the simplified approach proposed by Alexander does not work as well as variants of Riskmetrics using full information on interest rates.

In section 2 we describe the data used in the paper and present initial properties of univariate estimates of conditional volatilities across the term structure. Principal components are used in section 3 to reduce the dimensionality of the vector of volatilities across the term structure. As an alternative, in section 4 we use a factor model for daily interest rates changes to estimate conditional volatility at the specific maturities we consider. The ability of both approaches to account for volatility across the term structure of interest rates is compared in section 5. The differences between them when estimating VaR are presented in section 6. We close with the main conclusions in section 7.

2. THE DATA

We use closing daily prices from the secondary market for Spanish government debt to obtain daily estimates of the Nelson-Siegel discount rate model, from which zero-coupon rates can be inferred for any maturity. For simplicity, we limit our comparison to 1-, 3-, 6-, 8-, and 10-month rates, together with 1-, 3-, 5-, 6-, 7-, 8-, 9-, and 10-year rates. Our sample runs from September 1st, 1995 to December 31st, 2002.

Since January 1999, when the European Monetary Union was created, the European Central Bank, in coordination with central banks, has been in charge of implementing monetary policy in all country members, including Spain. Before that, Banco de España was the single official organism in charge of monetary police in Spain. Over the sample period considered, not only the institution in charge of monetary police, but also the way how policy is implemented, have changed. It is then almost mandatory to perform the common factor study in two different subsamples. The first sample covers from September 1st, 1995 to December 31st, 1998, the pre-monetary union period, while the second sample runs from January 4th, 1999 to December 31st, 2002.

Having obtained significant evidence from Engle and Ng [1993] tests[1] on the fact that good and bad news have a different impact on conditional volatilities of interest rates, we have used the EGARCH class of models to represent the conditional volatility of individual interest rates. The modelis specified for daily changes in interest rates because of their clear nonstarionarity. After running some specification tests, an EGARCH (1,1) model can be shown to adequately represent the conditional volatility in both subsamples. The estimated conditional volatility of the 1-month rate shows a high correlation with the volatilities of the 3-, 6-, 8-, 10-month and 1-year interest rates in both subsamples, while the conditional volatility of the 10-year rate displays a large correlation with the volatility of the 3-, 5-, 6-, 7-, 8-, 9-year rates of interest. Hence, it looks as if there is substantial volatility transmission across adjacent maturities, whereas transmission of volatility between the two ends of the term structure is much less obvious. In addition, the central region, represented by the one year maturity, seems to display some specific properties. This preliminary evidence suggests the difficulty of achieving a good representation of volatility across the term structure with just two factors, so that at least three factors might be needed. Exploring that possibility is the object of the next section.

1 The results of these tests are not shown to save space, but they can be obtained from the authors upon request.

3. A PRINCIPAL COMPONENT ANALYSIS OF VOLATILITIES ALONG THE TERM STRUCTURE

In an attempt to reduce the dimensionality of the vector of 13 time series of univariate conditional volatilities, we compute their principal components. The first five eigenvalues of the variance-covariance matrix of conditional volatilities in the first sample explain a percent cumulative explained variance of 63.53%, 90.98%, 96.84%, 97.89% and 99.35%. In consistency with our remarks in the previous section, three principal components would be enough to capture 95% of the time variation in the conditional volatilities, while up to five principal components would be needed to capture 99% of the time variation. The explanatory ability of the first three principal components increases clearly in the second sample, in which cumulative explained percent variance is: 82.99%, 92.86%, 97.46%, 98.98% and 99.54%. In this case, the first four factors capture 99% of the time variation in volatility, although again, three of them would be enough to capture 95% of the variation in the whole set of volatility time series.

Table 1 shows that, for the first sample, the coefficients defining the first principal component of the vector of univariate conditional volatilities capture volatility over the whole term structure, but with heavier weights for the shorter maturities. It is more an average level of volatility over the shorter end of the curve than a general level of volatility. The second component is represented with coefficients of opposite sign at both ends of the term structure. Even though the coefficients change somewhat for the different maturities, this component can be interpreted as representing the difference between the levels of volatility between the two ends of the term structure. In that sense, it can be interpreted as the slope of the term structure of volatilities. The loadings of the long term volatilities in the composition of the third principal component are almost zero, so that this component is represented as a linear combination of volatilities in the shorter end of the term structure. Because of the signs of the different coefficients, changes in this third

Table 1

Principal components for conditional volatilities

Sample: September 1995 to December 1998	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.52	0.44	0.38	0.27	0.17	0.15	0.22	0.22	0.20	0.18	0.18	0.18	0.17
Second principal component	-0.20	-0.25	-0.32	-0.25	-0.13	-0.03	0.32	0.34	0.32	0.31	0.31	0.31	0.32
Third principal component	0.58	0.28	-0.32	-0.49	-0.37	-0.30	-0.08	-0.06	-0.05	-0.05	-0.04	-0.02	0.00

Sample: January 1999 to December 2002	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.52	0.55	0.43	0.33	0.27	0.22	0.03	0.03	0.04	0.04	0.04	0.04	0.03
Second principal component	-0.12	-0.14	-0.02	0.04	0.08	0.09	0.43	0.46	0.41	0.36	0.32	0.29	0.27
Third principal component	0.64	0.16	-0.29	-0.36	-0.40	-0.41	0.03	0.08	0.08	0.07	0.07	0.07	0.08

Note: The table shows the weight of each volatility in the composition of each principal component.

component would imply changes of different sign in the volatilities of the 1-, 3-, 6- month rates, relative to changes in the volatility of the 8-, 10-, month and 1 year rates. This third component could be interpreted as representing changes in the curvature at the shorter end of the term structure of volatilities.[2]

Results in the second subsample are similar regarding the first and third component, while the second component can now be seen as representing the general level of volatility in the long end of the term structure. The estimates of the first two components point out to a different evolution of volatility at both ends of the term structure, at a diference of what happened over the first subsample.

Figures 1(a) to 4(a) present the conditional volatilities estimated from univariate EGARCH (1,1) models for 1-, 3-, 5- and 10-year maturities in the first sample, together with the conditional volatilities obtained for those maturities from the factor model for interest rate volatilities. Figures 1(b) to 4(b) present a scatter graph of the two conditional volatility estimates, showing a quite acceptable similarity.

To evaluate the ability of the first three principal components to account for the conditional volatility at each of the 13 maturities considered, we use the first three estimated principal components as explanatory variables in a system of regression equations having alternatively the volatility at each maturity as the dependent variable. We will refer to this system as the *factor model for interest rate volatilities*. In the first subsample, the regression R-square is above 95% for most maturities (table 2).[3] The ability of the first three components to explain the volatility of the 10 month, 1- and 3 years interest rate is a little lower. The fit in the second subsample is very similar, although the explanatory power for the 3-, 9- and 10 year interest rates is now somewhat lower. A good fit at this longer end of the term structure of volatilities seems to require an additional principal component (see figures 5(a,b) to 8(a,b)).

Mean Absolute Errors for the linear projections of volatility on the first three components are very low in each of the two subsamples and for each of the 13 maturities considered, being below 0.30 in all cases. With only a few exceptions, Percent Root Mean Square Errors (RMSE) in table 2 are below 5% in the first sample, reflecting the fact that the three first principal components explain, on average, 95% of the fluctuation in volatility over the term structure. RMSE values increase up to around 10% for the 10 month, 1- and 3 year maturities. RMSE values are a bit higher in the second sample but, on the other hand, they remain below 10% for all maturities.

In summary, we have shown that a relatively simple representation can account for the time behaviour of volatility over the term structure of interest rates. As a consequence, we can obtain volatility forecasts for a large set of interest rates at different maturities by forecasting just three variables, the first three principal components. The relevance of this result emerges from the need to use volatility forecasts in many questions regarding risk management as well as options trading in fixed income markets, even though it is not pursued in this paper.

2 These interpretations differ from the interpretation of level, slope and curvature that the first three principal components for interest rate levels usually receive in most fixed income markets.

3 Residuals in these regressions are stationary in all cases.

Figures 1 to 4

Factor model for conditional volatilities
September 1, 1995 to December 31, 1998

Figure 1(a). Conditional Standard Deviation
1 Year Interest Rate

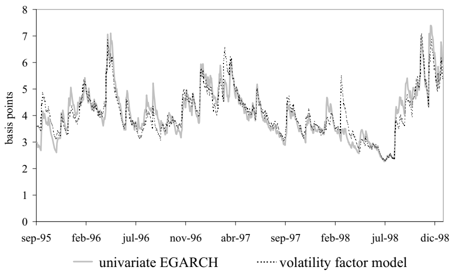


Figure 1(b). 1 year

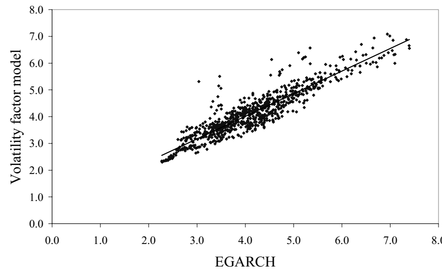


Figure 2(a). Conditional Standard Deviation
3 Year Interest Rate

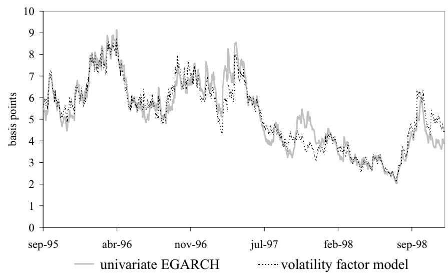


Figure 2(b). 3 year

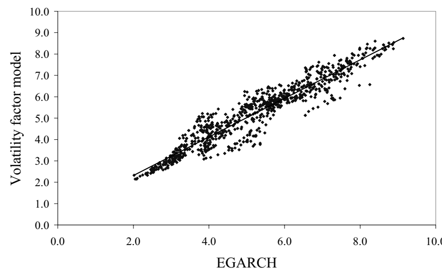


Figure 3(a). Conditional Standard Deviation
5 Year Interest Rate

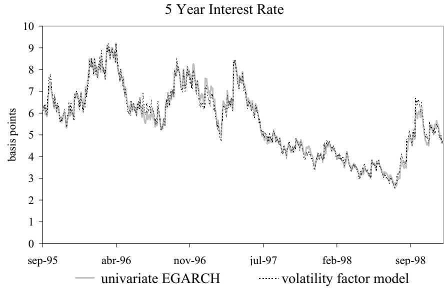


Figure 3(b). 5 year

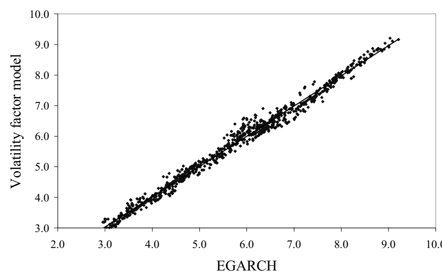


Figure 4(a). Conditional Standard Deviation
10 Year Interest Rate

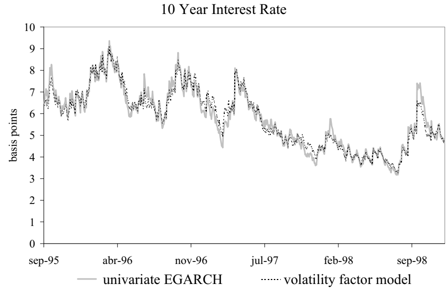
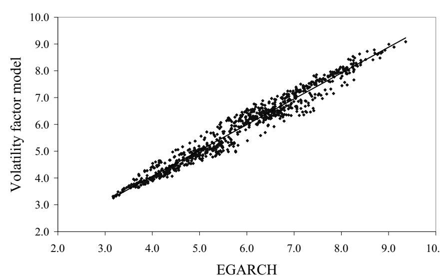


Figure 4(b). 10 year



Figures 5 to 8

Factor model for conditional volatilities
January 4, 1999 to December 31, 2002

Figure 5(a). Conditional Standard Deviation
1 Year Interest Rate

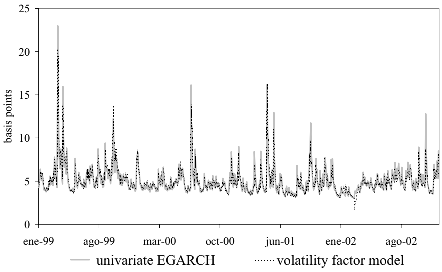


Figure 5(b). 1 year

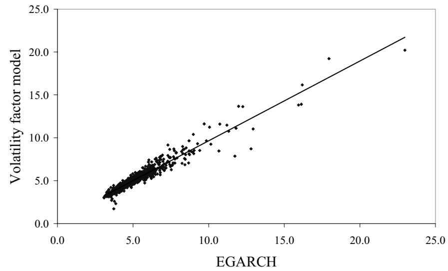


Figure 6(a). Conditional Standard Deviation
3 Year Interest Rate

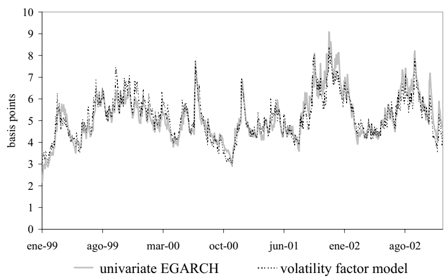


Figure 6(b). 3 year

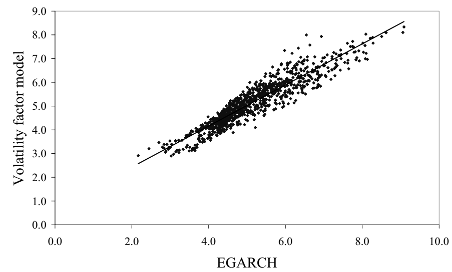


Figure 7(a). Conditional Standard Deviation
5 Year Interest Rate

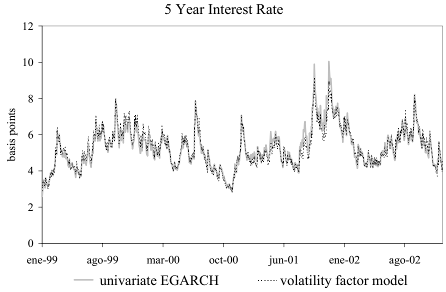


Figure 7(b). 5 year

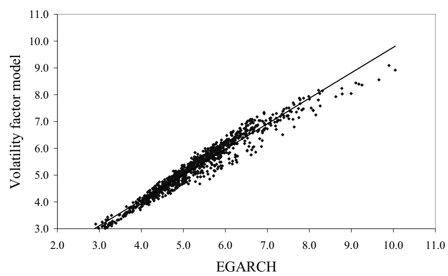


Figure 8(a).Conditional Standard Deviation
10 Year Interest Rate

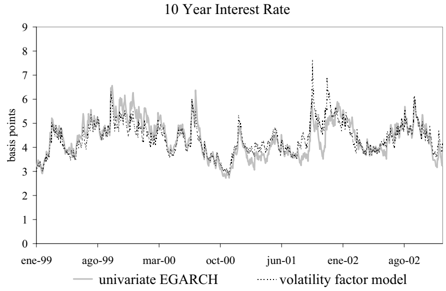


Figure 8(b). 10 year

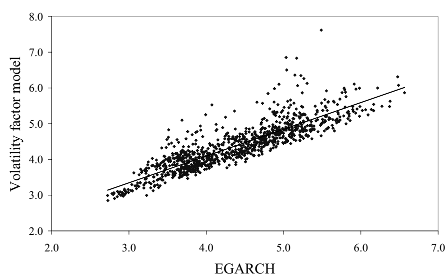


Table 2

Goodness of fit. Factor model for conditional volatilities

	Sample: September 1995 to December 1998												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Coefficient of determination (R^2)	0.99	0.97	0.98	0.99	0.86	0.84	0.90	0.99	0.99	0.99	0.99	0.98	0.96
Mean Absolute Error	0.17	0.17	0.23	0.13	0.31	0.28	0.37	0.13	0.09	0.09	0.12	0.16	0.20
Root Mean Square Error	4.7	4.1	6.3	4.9	8.8	9.7	10.3	3.1	2.1	2.2	2.9	3.7	4.7

	Sample: January 1999 to December 2002												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Coefficient of determination (R^2)	0.99	0.99	0.98	1.00	0.98	0.93	0.87	0.95	0.98	0.97	0.93	0.85	0.75
Mean Absolute Error *	0.27	0.28	0.24	0.09	0.18	0.27	0.31	0.18	0.11	0.11	0.17	0.22	0.29
Root Mean Square Error ++	5.7	8.1	7.0	2.3	4.4	6.7	7.4	4.2	2.7	2.9	4.6	6.8	8.7

Note: The table presents the coefficient of determination (R^2) for regression: $h_{j,t}^2 = \alpha_j^1 g_{1,t} + \phi_j^2 g_{2,t} + \phi_j^3 g_{3,t} + \varepsilon_t$, where $h_{j,t}^2$ represents the conditional variance of the interest rate at maturity *j*, and $g_{1,t}$, $g_{2,t}$, $g_{3,t}$ represent the first three principal components for the vector of conditional volatilities of interest rate changes at 1-, 2-, 3-, .., 10-year maturities.
(*)Absolute Error. (++) Percent Root Mean Square Error.

4. A PRINCIPAL COMPONENT ANALYSIS OF INTEREST RATE CHANGES

If we have a good model to account for the term structure of interest rates, it seems natural to think that this model should also be able to account for the behaviour of interest rate volatility. Such assumption underlines the work by Alexander (2000) and Gento (2001) among others. The question to explore is whether the volatility representation that emerges from a factor model for interest rate changes will be the same as the one we get from the factor model for interest rate volatilities we have described in the previous section.

To address this issue, we construct a factor model for interest rate changes through a principal component analysis, along the lines of Alexander (2000). So, the first part of this exercise is standard. The percent variance explained by the first principal component of daily interest rate changes in the first subsample is of 51.04%. The second and third components additionally explain, respectively, 40.54% and 5.80% of the variance, so that changes the first three components together explain more than 95% of daily changes along the term structure of interest rates.[4] At a difference of results obtained with the vector of univariate conditional volatilities in the previous section, now both subsamples produce very similar results. In the second sample the percent variance explained by the first component is 58.8%, while the percentage of variance explained by the second and third components is 35.2% and 4.16%, respectively. The percent cumulative variance explained by the first three components is in this case of 98.99%.

4 These figures cannot be compared to those of the previous section, since they refer to the space of daily interest rate changes, as opposed to the space of conditional volatilities in the previous section.

Table 3

Principal components for interest rate changes.

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.44	0.37	0.29	0.25	0.21	0.17	0.21	0.25	0.26	0.26	0.26	0.26	0.26
Second principal component	-0.52	-0.39	-0.22	-0.13	-0.04	0.03	0.25	0.27	0.27	0.27	0.27	0.27	0.27
Third principal component	0.40	0.09	-0.25	-0.39	-0.46	-0.50	-0.25	0.06	0.12	0.14	0.15	0.14	0.12

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.46	0.43	0.40	0.37	0.35	0.33	0.17	0.11	0.09	0.08	0.08	0.07	0.07
Second principal component	-0.20	-0.17	-0.11	-0.07	-0.03	0.01	0.32	0.40	0.40	0.38	0.36	0.34	0.32
Third principal component	0.55	0.26	-0.05	-0.21	-0.33	-0.42	-0.40	-0.05	0.06	0.13	0.18	0.20	0.22

Note: The table shows the weight of each interest rate in the composition of the first three principal components

Table 3 shows that, for the first sample, the coefficients defining the first principal component are quite similar over the whole term structure, so that this component can be seen to represent daily global shifts across the whole term structure of interest rates. The second component is characterized by coefficients of opposite sign at both ends of the term structure, so that this component can be interpreted as a slope component of interest rate changes. Finally, the third component can be interpreted as a curvature component. These results are fully in line with similar ones obtained for different international fixed income markets. Steeley [1990], Litterman and Scheinkman [1991], Kahn y Gulrajani [1993], D'Ecclesia and Zenios [1994], Navarro y Nave [1995], Barber and Copper [1996], Bliss [1997] are some examples.

The ability of the first three principal components to explain daily changes in interest rates can be examined by estimating:

$$dr_{j,t} = \sum_{i=1}^{3} \phi_i^j \, df_{i,t} + \varepsilon_{j,t} \tag{1}$$

where $dr_{j,t}$ represents daily changes in interest rate at the j -th maturity, for j = 1, 3-, 6-, 12-month, 1-, 3-, 5-, 6- and 10-years, and $df_{i,t}$, i =1, 2, 3, represents the first three principal components of daily changes in interest rates. The R-squared of the regression is quite high in all cases, being generally above 95% in both subsamples.[5] With a single exception, the Mean Absolute Error is below 1 basis point for all maturities in each of the two samples considered (Table 4).

Now, the point is how to move from the conditional volatilities for the principal components of daily interest rate changes, to conditional volatilities for the vector of interest rates. Following Alexander (2000), we estimate the variance-covariance matrix of the vector of interest rates by:

$$Var(dr_t) = AVar(df_t)A^T \tag{2}$$

5 Again, with stationary residuals

Table 4

Goodness of fit. Factor models for interest rate changes

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Coefficient of determination (R^2)	0.99	0.98	0.99	0.99	0.95	0.88	0.91	0.97	0.99	1.00	0.99	0.98	0.96
Mean Absolute Error (MAE)*	0.40	0.17	0.30	0.31	0.30	0.39	1.10	0.69	0.37	0.16	0.32	0.57	0.81

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Coefficient of determination (R^2)	0.99	1.00	1.00	0.99	0.99	0.99	0.95	0.96	0.98	1.00	0.99	0.95	0.89
Mean Absolute Error (MAE)*	0.49	0.13	0.18	0.28	0.31	0.30	0.64	0.65	0.40	0.10	0.27	0.59	0.88

Note: The table presents the coefficient of determination (R^2) for the regression: $dr_t(j) = \phi_j^1 f_{1,t} + \phi_j^2 f_{2,t} + \phi_j^3 f_{3,t} + \varepsilon_t$, for j=1, 2, 3, ..10 year, where $dr_t(j)$ represents the time-*t* change in interest rate at maturity (*j*) and $f_{1,t}$, $f_{2,t}$, $f_{3,t}$ are the first three principal components for the vector of interest rate changes at 1-, 2-, 3-, … 10-year maturities.
(*) Absolute errors.

where *Var(df$_t$)* is a diagonal matrix with the conditional variance of the first three principal components along the diagonal, and *A* is a 13 by 3 matrix having in each row the estimated coefficients from each individual regression[6] in (1). Then, *Var(dr$_t$)* is a 13 by 13 matrix representing the conditional variance-covariance matrix of interest rates.

To Engle and Ng [1993] tests[7] again show evidence that good and bad news have different impact on the conditional volatility of the principal components, so we decided to use again the EGARCH specification to represent their individual conditional volatility. Afterwards, expression (2) is used to produce conditional variance and covariances for the 13 interest rates considered.

As we did in the previous section, we compare the conditional variance estimates obtained from this procedure with those obtained from a univariate EGARCH (1,1) specification for each interest rate. Time fluctuations in the conditional variances obtained from the factor model for interest rate changes are similar to those experienced by univariate EGARCH conditional variances, as shown by correlation coefficients between daily changes in volatilities estimated from the two approaches. A comparison between Figures 1(a,b)-4(a,b) with Figures 9(a,b)-12(a,b), and Figures 5(a,b)-8(a,b) with Figures 13(a,b)-16(a,b) clearly shows that the conditional variances obtained from a factor model for daily interest rate changes do not fit univariate EGARCH conditional variances as well as those obtained from a factor model for interest rate volatilities. That the factor model for conditional volatilities should produce a good fit is not surprising. The relevant result is that the conditional volatilities obtained from the factor model for interest rate changes through (2) do not fit univariate EGARCH variances so nicely.

6 Coefficients in matrix *A* are the loadings of each interest rate in each principal component. So, in fact, it is not necessary to estimate equation (1) to get the variance-covariance matrix of a large set of interest rates, once the principal component analysis has been done.

7 Not shown in the paper, for simplicity.

Figures 9 to 12

Factor model for interest rate changes
September 1, 1995 to December 31, 1998

Figure 9(a). Conditional Standard Deviation
1 Year Interest Rate

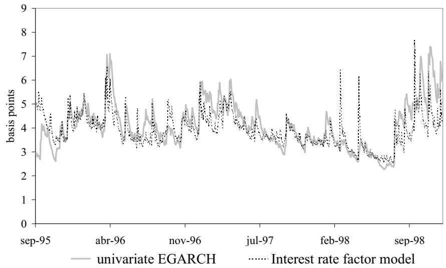


Figure 9(b). 1 year

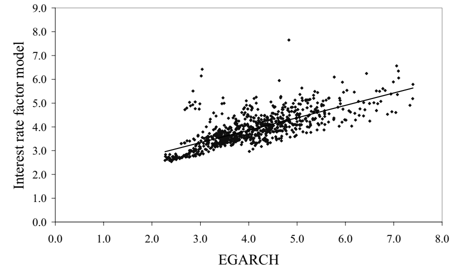


Figure 10(a). Conditional Standard Deviation
3 Year Interest Rate

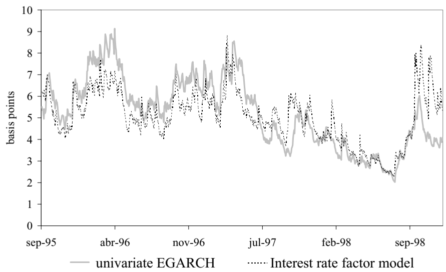


Figure 10(b). 3 year

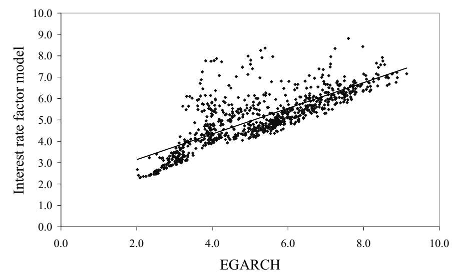


Figure 11(a). Conditional Standard Deviation
5 Year Interest Rate

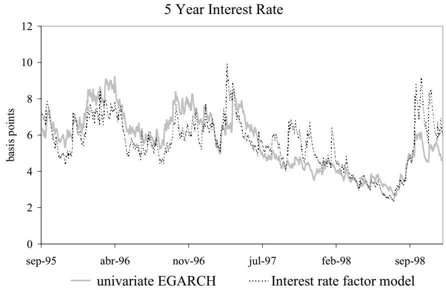


Figure 11(b). 5 year

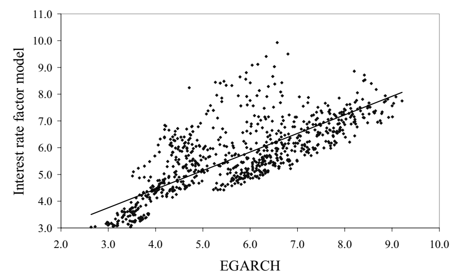


Figure 12(a). Conditional Standard Deviation
10 Year Interest Rate

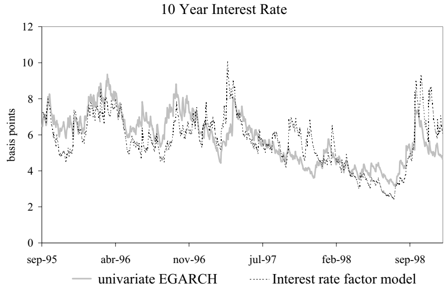
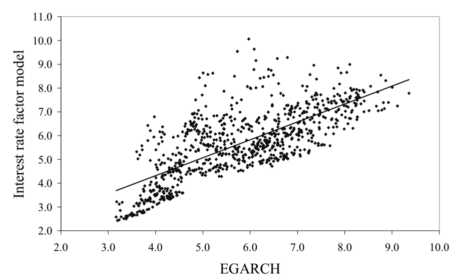


Figure 12(b). 10 year



Figures 13 to 16

Factor model for interest rate changes
January 4, 1999 to December 31, 2002

Figure 13(a). Conditional Standard Deviation
1 Year Interest Rate

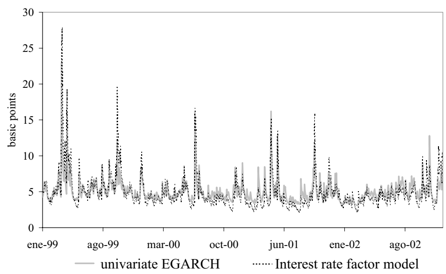


Figure 13(b). 1 year

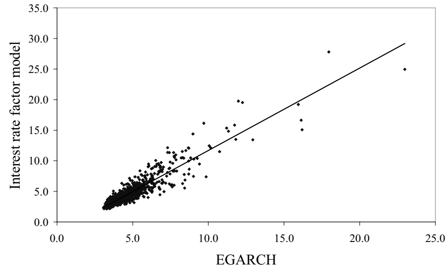


Figure 14(a). Conditional Standard Deviation
3 Year Interest Rate

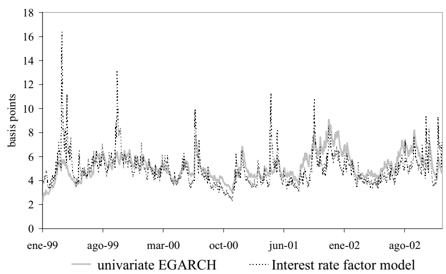


Figure 14(b). 3 year

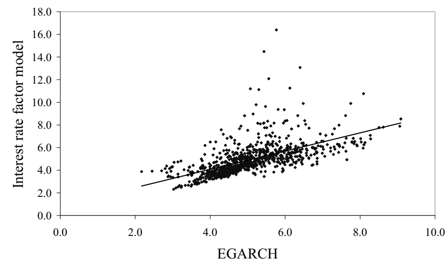


Figure 15(a). Conditional Standard Deviation
5 Year Interest Rate

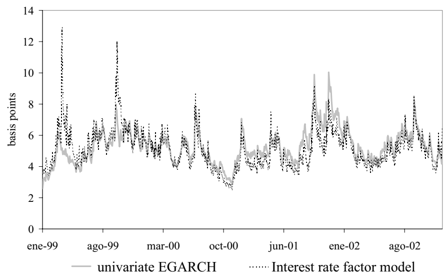


Figure 15(b). 5 year

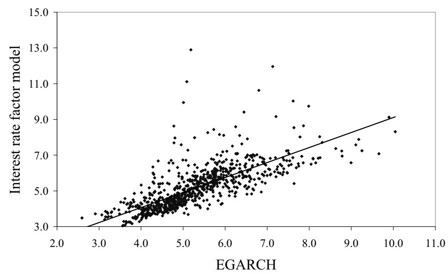


Figure 16(a). Conditional Standard Deviation
10 Year Interest Rate

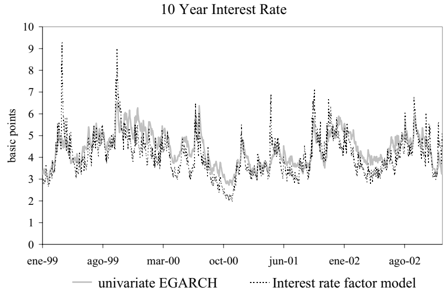
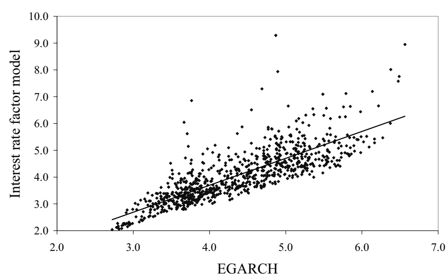


Figure 16(b). 10 year



5. DO FACTOR MODELS FOR INTEREST RATE VOLATILITIES AND FOR INTEREST RATE CHANGES LEAD TO THE SAME VOLATILITY ESTIMATES?

Table 5 presents Percent Root Mean Square Errors (RMSE) and Mean Absolute Errors (MAE) for both factor models: the one for interest rate volatilities, and the one for interest rate changes, considered as alternative estimates of univariate conditional variances. In the first subsample, the RMSE for the factor model of volatilities remains below 5% for most maturities, while it falls above 16% at all maturities for the factor model for interest rate changes. In the second subsample, RMSE for the factor model for interest rate changes is more than three times the RMSE for the factor model for volatilities, for most maturities. In each of the two samples and for all maturities the Mean Absolute Error is higher for the factor model for interest rate changes. These results show that the volatility representation obtained from a factor model for interest rate volatilities is different from the one obtained from a factor model for interest rate changes, the first one fitting much better the conditional variances estimated from univariate EGARCH models.

Table 5
Conditional Volatilities. Goodness of fit from both factor models

	Sample: September 1995 to December 1998												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Volatility factor model													
- RMSE	4.7	4.1	6.3	4.9	8.8	9.7	10.3	3.1	2.1	2.2	2.9	3.7	4.7
- MAE	0.2	0.2	0.2	0.1	0.3	0.3	0.4	0.1	0.1	0.1	0.1	0.2	0.2
Interest rate factor model													
- RMSE	17.7	20.0	34.2	30.4	18.0	16.5	21.0	18.8	18.4	18.5	18.6	18.8	19.4
- MAE	1.3	1.1	1.2	1.0	0.6	0.5	0.8	0.8	0.8	0.8	0.8	0.8	0.8

	Sample: January 1999 to December 2002												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Volatility factor model													
- RMSE	5.7	8.1	7.0	2.3	4.4	6.7	7.4	4.2	2.7	2.9	4.6	6.8	8.7
- MAE	0.5	0.1	0.2	0.3	0.3	0.3	0.6	0.7	0.4	0.1	0.3	0.6	0.9
Interest rate factor model													
- RMSE	31.2	34.2	15.9	11.6	15.1	19.0	22.4	15.2	12.3	10.6	10.7	12.2	14.9
- MAE	1.4	1.1	0.5	0.5	0.6	0.8	0.8	0.6	0.4	0.4	0.4	0.4	0.5

Note: Error statistics for the comparison between the conditional volatilities estimated from univariate EGARCH models for interest rate changes, with the conditional volatilities obtained from both factor models.

RMSE: Percent root mean square error.
MAE: Mean of absolute errors.

Linear correlation coefficients between both sets of conditional variance estimates are not too large for the first subsample, falling between .70 and .80. They increase to between .85 and .95 over the second subsample, for most maturities. It looks as if the European monetary union not only introduced a tighter link between interest rate levels, but also between their volatilities. Sample mean and median across the term structure are similar for large maturities, but they show differences of up to one percent point for the shorter maturities. But the main difference relates to the leptokurtosis in the estimates from the factor model for volatilities, as opposed to a slight platykurtosis of the estimates obtained from the factor model for interest changes in both subsamples. This is relevant, and it also shows in the range obtained with the 10% and 90% percentiles. On the contrary, the sample and interquartilic ranges are similar for both estimates at all maturities, so the differences between the two sets of estimates arise at extreme volatility levels. The factor model for interest rate changes seems specially unable to fully capture the very hgh levels of volatility, producing a difference between the frequency distributions from both estimates, whose equality we now proceed to test formally.

Mann-Whitney and Kruskal-Wallis statistics to test for whether both volatility series have the same mean are shown in table 6. They display widespread evidence against the null hypothesis of homogeneity, specialy in the second subsample. The Siegel-Tuckey statistic to test for whether the conditional volatilities produced by the two models have similar variation is

Table 6
Tests for equal conditional variances

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Mann-Whitney	2.56	2.54	5.31	5.91	1.99	4.14	2.03	0.52	0.06	0.28	0.06	0.55	1.48
	(0.01)	(0.01)	(0.00)	(0.00)	(0.05)	(0.00)	(0.04)	(0.60)	(0.95)	(0.78)	(0.95)	(0.58)	(0.14)
Kruskal-Wallis	6.53	6.43	28.16	34.88	3.97	17.15	4.12	0.27	0.00	0.08	0.00	0.31	2.18
	(0.01)	(0.01)	(0.00)	(0.00)	(0.05)	(0.00)	(0.04)	(0.60)	(0.95)	(0.78)	(0.95)	(0.58)	(0.14)
Siegel-Tukey	0.81	5.01	82.86	107.2	25.18	11.64	60.98	30.94	10.27	5.06	4.00	4.67	7.11
	(0.37)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.05)	(0.03)	(0.01)
Wilcoxon	3.17	1.58	5.74	5.98	0.08	-11.61	-5.25	-3.66	-3.34	-3.24	-3.78	-5.02	-6.72
Kolmogorov-Smirnov	0.07	0.08	0.20	0.21	0.10	0.10	0.12	0.08	0.07	0.07	0.06	0.07	0.08

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Mann-Whitney	9.90	9.83	4.65	0.60	0.60	3.07	6.39	3.74	2.75	2.49	3.33	5.57	9.23
	(0.00)	(0.00)	(0.00)	(0.55)	(0.55)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
Kruskal-Wallis	97.95	96.53	21.63	0.36	9.48	40.84	46.84	13.97	7.58	6.19	11.09	31.01	85.21
	(0.00)	(0.00)	(0.00)	(0.55)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
Siegel-Tukey	43.43	53.82	1.36	11.74	47.92	114.02	28.61	17.63	27.18	40.30	60.26	82.58	104.92
	(0.00)	(0.00)	(0.24)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Wilcoxon	17.34	19.31	16.85	5.951	-5.735	-10.26	-13.99	-13.45	-11.47	-10.05	-11.25	-15.33	-19.72
Kolmogorov-Smirnov	0.246	0.236	0.1	0.047	0.112	0.206	0.152	0.103	0.091	0.102	0.119	0.156	0.239

Note: p-values in parenthesis. Critical values for the Kolmogorov test are 0.050, 0.061 and 0.073 at 90%, 95% and 99% confidence levels. Critical values for Wilcoxon test are 1.29, 1.65 and 2.30 at 90%, 95% and 99% confidence levels.

presented in the same table, offering again strong evidence against the null hypothesis. Finally, the Wilcoxon and the Kolmogorov-Smirnov statistics in table 6 test whether the probability distributions for the conditional volatility estimates obtained from both factor models are the same. In both subsamples and for all maturities, the two statistics offer evidence against the null hypothesis.

Summarizing, the conditional volatility estimates produced by the factor model for interest rate volatilities and by the factor model for interest rate changes display statistically significant differences, being the first approach, as it should be expected, the one that better fits the univariate EGARCH volatilities. Hence, to estimate volatilities over the term structure, we might be better off by using a volatility factor model than an interest rate factor model. That runs against the increasingly standard practice of estimating conditional volatilities over the term structure from the factor representation for interest rate changes, and suggests that reducing the information contained in a vector interest rate in a small number of factors may be inappropriate for most risk management purposes. To check that possibility, in the next section we analyze the ability of both factor models for risk evaluation of fixed-income assets.

6. IMPLICATIONS OF DIMENSIONALITY REDUCTION FOR VALUE AT RISK ESTIMATION

We have seen in the previous section how the simplification embedded in the factor model for interest rates leaves aside significant information relative to the estimation of conditional variances, which should be expected to have significant implications for risk management in fixed income markets. Given its relevance as a risk indicator, we focus in this section on Value at Risk (VaR) estimation[8] as it is usually performed in the financial industry,

Changes in the price of a bond can be approximated by a vector of durations applied to the vector of changes in the relevant interest rates. That, in turn, allows us to estimate the conditional variance of changes in bond prices and, by extension, the conditional variance of changes in the market value of any fixed income portfolio. Under the assumption of Normality for daily interest rate changes, we can then easily estimate VaR. Even though the Normality assumption is rejected by the data, it is still used by most private and public financial institutions, and it is used as a benchmark in some more sophisticated academic exercises. Furthermore, some recently published work points out to the fact that, at least during stable periods, the Normality assumption may overperform the usual alternatives [see Yong et al. (2006), Danielsson (2002), Chang 2004, Sarma et al. (2003)].

Under continuous time discount, the theoretical price for a bond maturing *k*-periods from now can be written,

8 The Value at Risk (VaR) of a portfolio gives us the maximum amount that an investor may lose over a given time horizon, with a given probability. Formally, the VaR of a portfolio at 1-a% confidence level is the 1-a% percentil of the probability distribution of changes in portfolio value. Different methodologies have been developed to estimate the VaR of a portfolio, like Montecarlo simulation, historical simulation, parametric method also called variance-covariance approach and more recently, Extreme Value Theory (ETV).

$$p_t^k = \sum_{m=1}^{k-1} (cN)\exp(-mr_t(m)) + \left[(c+1)N\exp(-kr_t(k))\right] \tag{3}$$

with N denoting the face value of the bond, c the coupon, $r_t(m)$ the time-t zero coupon interest rate at maturity m. From (3), we can approximate price changes through,

$$dp_t^k \approx D_{k,t}\, dr_t \tag{4}$$

where:

$$D_{k,t} = \begin{bmatrix} \frac{\partial p_t^k}{\partial r_t(1)} & \frac{\partial p_t^k}{\partial r_t(2)} & \frac{\partial p_t^k}{\partial r_t(3)} & ,..., & \frac{\partial p_t^k}{\partial r_t(k)} \end{bmatrix}, \text{ and } dr_t = \begin{bmatrix} dr_t(1) \\ dr_t(2) \\ dr_t(3) \\ .... \\ dr_t(k) \end{bmatrix}.$$

Taking variances at both sides of equation (4) we get:

$$\sigma^2_{dp_t^k} \approx D_{k,t} \Sigma_{dr_t} D'_{k,t} \tag{5}$$

where $\sigma^2_{dp_t^k}$ represents the conditional variance of price changes in the k -th bond, and Σ_{dr_t} represents the conditional variance-covariance matrix of the set of interest rates determining the theoretical price of bond k.

We can now see the difference between using the two different approaches to estimate the conditional variance-covariance matrix, Σ_{dr_t}. The factor model for interest rate changes provides us with a representation,

$$dr_t \approx AF_t \tag{6}$$

where $F_t = (f_{1,t}, f_{2,t}, f_{3,t})$ is the 3 x 1 vector made up with the first three principal components of the vector of interest rate changes $dr_t = [dr_t(1), dr_t(2), dr_t(3), ..., dr_t(n)]$, and A is a k x 3 matrix having as columns the eigenvectors associated to the three largest eigenvalues of the variance-covariance matrix of interest rate changes:

$$A = \begin{bmatrix} a_{f_1}^1 & a_{f_2}^1 & a_{f_3}^1 \\ a_{f_1}^2 & a_{f_2}^2 & a_{f_3}^2 \\ a_{f_1}^3 & a_{f_2}^3 & a_{f_3}^3 \\ .. & .. & .. \\ a_{f_1}^k & a_{f_2}^k & a_{f_3}^k \end{bmatrix}$$

Taking variances at both sides of equation (6) we get an estimate of Σ_{dr_t}:

$$\Sigma_{dr_t} \approx A\Omega_t A' \tag{5}$$

with Ω_t being the variance-covariance matrix of the first three principal components of the vector of interest rate changes:

$$\Omega_t = \begin{bmatrix} \sigma^2_{f_{1,t}} & 0 & 0 \\ 0 & \sigma^2_{f_{2,t}} & 0 \\ 0 & 0 & \sigma^2_{f_{3,t}} \end{bmatrix}$$

The main virtue of this approach is to achieve an important reduction of dimensionality in the estimation of the variance-covariance matrix of the vector of relevant interest rates. We just need to estimate 3 second order moments for a vector of k interest rates, instead of having to estimate $k(k+1)/2$ variances and covariances.

On the other hand, the volatility factor model starts precisely from univariate estimates of the principal diagonal of the Σ_{dr_t} matrix. Since the volatility factor model does not allow us to obtain covariances betwen interest rates at different maturities, we estimate the covariances between any two interest rates from an exponentially weighted moving average model with $\lambda = .94$ and a 74 days window.[9] This approach maintains the original dimension of the interest rate vector, having computational simplicity as its main advantage. This second approach to computation of the conditional variance-covariance is very similar to Riskmetrics, from which it departs only by using an EGARCH univariate specification for estimation of conditional variances.[10]

The two implied estimates of the conditional variance of changes in bond prices can, in turn, be used in (4) to compute the VaR for any given bond under the assumption of Normality. We proceed to estimateVaR for each of 4 bonds with 1-, 3-, 5- and 10-year maturities paying an annual 5% coupon, at 95% and 99% confidence levels, and a one-day horizon. We then examine actual daily price changes in the theoretical bonds, as implied by actual daily fluctuations in zero coupon interest rates in our sample, and compare them with VaR estimates. If the estimation of the theoretical VaR is appropriate, we should expect about 95% and 99% of daily price changes to fall below the VaR estimates. Our first sample being of size 813 data points, that amounts to 41 daily price changes below the 95% VaR and 8 daily price changes below 99% VaR. The size of the second sample considered is 992, so that about 50 daily price changes should be below the 95% VaR and 10 daily price change below 99% VaR.

Table 7 shows the absolute and relative frequencies of price changes below the 95% and 99% VaR for each bond. In the first sample, we observe between 31 and 38 daily price changes below the 95% VaR with the factor model for volatilities, and between 26 and 32 daily price changes for the factor model for interest rate changes. This amounts to a percentage between 3.9% and 4.7% under the volatility factor model, and between 3.2% and 3.9% under the factor model for interest rate changes. Hence, the 95% VaR obtained from both methods is overestimating risk, since the number of days that prices change by less than the 95% VaR is below its theoretical level, but the

9 The same parameters used by Riskmetrics.

10 Riskmetrics uses an exponentially weighted average scheme similar to the one we use for estimating conditional covariances.

Table 7
Number and percentage (in brackets) of exceptions

	Volatility factor model				Interest rate factor model			
	Bond at maturity				Bond at maturity			
	1 Year	3 Year	5 Year	10 Year	1 Year	3 Year	5 Year	10 Year
September 1995 to December 1998								
VaR (5%)	32*	38	31	35	32	31	31	26
	(3.9)+	(4.7)	(3.8)	(4.3)	(3.9)	(3.8)	(3.8)	(3.2)
VaR (1%)	10	9	8	16	11	10	9	10
	(1.2)	(1.1)	(1.0)	(2.0)	(1.3)	(1.2)	(1.1)	(1.2)
January 1999 to December 2002								
VaR (5%)	34*	48	50	54	37	46	43	32
	(3.4)+	(4.8)	(5.0)	(5.4)	(3.7)	(4.6)	(4.3)	(3.2)
VaR (1%)	16	23	10	11	12	18	12	6
	(1.6)	(2.3)	(1.0)	(1.1)	(1.2)	(1.8)	(1.2)	(0.6)

Note: (*) Number of exceptions. The number of exceptions in a sample of *n* data is the number of times the price change of the bond *j* (for *j*= 1 year, 3 year, 5 year and 10 year) falls below VaR(α %).
(+) Percentage of exceptions.

bias in the volatility factor model is smaller. The 99% VaR estimate seems to perform somewhat better, approaching the 99% theorical confidence level in both models. At this confidence level both VaR models underestimate risk, with no model performing better than the other in estimating VaR.

Results for the second sample are similar to those for the first one. Between 34 and 54 daily price changes fall below the 95% VaR with the factor model for volatilities, and between 32 and 46 daily price changes for the factor model for interest rate changes. These amount to a percentage between 3.4% and 5.4% under the volatility factor model, and between 3.2% and 4.6% under the factor model for interest rate changes. At the 99% confidence level, we observe between 11 and 23 daily price changes below the estimated VaR with the factor model for volatilities, and between 6 and 18 daily price changes for the factor model for interest rate changes. This amounts to a percentage between 1.0% and 2.3% under the volatility factor model, and between 0.6% and 1.8% under the factor model for interest rate changes. Once again, we find that at 95% confidence level the VaR estimate from the volatility factor model performs better than the VaR estimate from the factor model for interest rate changes, while at 99% confidence level none of them seems to perform better than the other in estimating VaR.[11]

11 In order to formally test whether VaR estimates are accurate, a test proposed by Kupiec (1995) defines a random variable x taking the value 1 if the portfolio value changes below the estimated VaR(α%), and 0 otherwise. Under the hypothesis that any given occurrence (either exception or not exception) does not help predicting the outcome of the next day, the number of exceptions in a sample of *n* data, *i.e.*, the number of days that the change in the portfolio value falls below VaR(α%) is distributed as a binomial (n,p) with $p=\alpha$. After building a confidence interval using this distribution, we reject the null hypothesis that $p = \alpha$ if the number of exceptions is out of that confidence interval, concluding that the VaR estimate is inaccurate. The opposite will happen if the number of exceptions is inside the interval. In both samples, and for the two models considered, the number of exceptions falls inside the 95% and the 99% confidence levels in most bonds, suggesting that VaR estimates we get from both models are statistically accurate. But, given our sample evidence, we feel that this statistical result just shows the low power of this test.

The two approaches we have followed in this section to compute VaR display two significant differences: i) the models used to obtain conditional variances and covariances, and ii) the information set they use. Under Alexander (2000) suggestion, conditional variances and covariances for the vector of interest rates are obtained from univariate EGARCH models for each of the three principal components for interest changes. Under the alternative approach, EGARCH models are used to estimate conditional variances for each interest rate affecting the portfolio's value, conditional covariances being estimated through the same exponential weighting moving average model used by Riskmetrics. On the other hand, Alexander's approach starts by collapsing the information in the whole vector of interest rates to the information set contained in just the three principal components, while the alternative approach works with the whole information set, as it is also the case with the Riskmetrics methodology. Our results suggest that the simplification in Alexander's approach may hinder its more elaborate approach to modelling the conditional variance-covariance matrix of interest rate changes.

The alternative approach we have opposed to Alexander's factor model is not too different from Riskmetrics methodology. In fact, the models used to estimate conditional covariances are the same, the only difference being our use of EGARCH models for conditional variances for each interest rate affecting the portfolio value. We have also run some comparisons between the two methodologies using the same sample data as above, with the result that VaR performance is so similar that it does not seem to pay to use EGARCH models, the full Riskmetrics approach leading to a good enough performance in VaR estimation.

7. CONCLUSIONS

A linear factor representation for the term structure of interest rates through the use of principal components leads to a natural representation for volatility across the term structure. An alternative way to represent volatility across the term structure of interest rates is by means of a factor model for the conditional volatilities obtained from univariate interest rate models. Contrary to what might be expected, we have shown that these two approaches lead to statistically significant differences in conditional volatility estimates, an empirical result with clear implications for risk management in fixed income markets.

We have focused in this paper in the possible implications for Value at Risk estimation under both approaches, using 5%-coupon theoretical bonds at 1-, 3-, 5- and 10-year maturities. At the 99% confidence level, neither model seems to perform better than the other in estimating VaR at the different maturities. However, at the 95% confidence level, VaR estimates from the volatility factor model work better than those from the interest rate factor model, which has usually been proposed in the literature.

The volatility factor model we have compared as an alternative is not too different from the Riskmetrics standard regarding VaR estimation, since they both use the same estimates for conditional covariances between pairs of interest rates. In fact, both lead to very similar VaR estimates. So, our results seems to suggest that the information reduction implied by the simplification involved in the interest rate factor model leaves out relevant information regarding volatility across the term structure, which is otherwise taken into account in methods that, like Riskmetrics, work on the full set of interest rates. Furthermore, the information loss is not compensated by the more elaborate approach to model conditional variances.

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