Financial innovation and arbitrage in the Spanish bond market

Innovación financiera y arbitraje en el Mercado de Bonos Español

Susana López[1,2,3]
Universidad Autónoma de Madrid.
Departamento de Análisis Económico: Economía Cuantitativa

Resumen: El artículo analiza empíricamente el nivel de arbitraje secuencial en el Mercado de Bonos español. El estudio se realiza con bonos libres de riesgo y de opción implícita del gobierno Español. Este hecho establece una clara distinción entre esté trabajo y la literatura empírica relacionada ya que no hay bonos con riesgo o activos derivados que afecten a nuestro análisis. Como consecuencia, la ausencia de arbitraje secuencial es equivalente a la existencia de una estructura temporal de tipos de interés que iguala el precio del conjunto total de bonos que proporciona el Banco de España. Por tanto, la principales conclusiones parecen ser robustas debido a que solo dependen de hipótesis generales y sencillas. Los resultados del análisis empírico pueden resultar útiles a gestores de carteras e investigadores ya que parecen revelar existencia de arbitraje secuencial. Además, el número de oportunidades de arbitraje se incremente significativamente en 1998, momento en el que importantes innovaciones financieras se llevaron a cabo, y entre otras nuevas posibilidades, los agentes comenzaron a negociar tanto el bono en su totalidad como sus cupones por separado. La inexperiencia asociada a la innovación financiera podría haber dado lugar a ineficiencias en el mercado.

Palabras clave: Arbitraje secuencial, mercado de bonos, innovación financiera, criterio mínimas.

Código JEL: G11, G12, G14.

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Abstract: This paper empirically tests the level of sequential arbitrage in the Spanish bond market. The test is implemented by drawing on default free and option free pure discount and coupon bonds issued by the Spanish government. This fact seems to be a clear distinction between this paper and the related empirical literature since there are no risky bonds or derivative securities involved in our analysis. As a consequence, the sequential arbitrage absence is just equivalent to the existence of a term structure of interest rates matching the whole set of bond prices as provided by The Bank of Spain. Thus, the main conclusions seem to be robust because they only depend on very general and simple hypotheses and, particularly, no dynamic assumptions are required. The results of the empirical analysis may be useful to managers, traders and researchers since it seems to reveal the existence of sequential arbitrage. Furthermore, the number of arbitrage opportunities significantly increased in 1998, when important innovations were implemented and, amongst other new possibilities, agents began trading each whole bond and its coupons (strips) separately. The inexperience associated with financial innovations may lead to inefficiencies in the market.

Key words: Sequential arbitrage, bond markets, financial innovation, minimax criteria

JEL Classification: G11, G12, G14

1. INTRODUCTION

The absence of arbitrage opportunities, or sequential arbitrage opportunities in the case of bond markets, is a basic assumption very common in financial literature. The existence of arbitrage is hardly compatible with any kind of equilibrium and, therefore, no arbitrage conditions are almost always assumed in every classical theoretical approach concerning asset pricing or asset allocation.

However the empirical evidence seems to reveal that the arbitrage may occur in practice (see for instance Lee and Nayar (1993), Chen and Knez (1995), Kamara and Miller (1995), Kempf and Korn (1998), Balbás *et al.* (2000) and Grinblatt and Longstaff (2000)) and this fact has motivated several authors to introduce new viewpoints. A first line of research focuses on market imperfections because they can make the arbitrage be apparent but not real. So, Jouini and Kallal (1995) present a general treatment of transaction costs and their influence on classic theoretical results concerning arbitrage free models, Schleifer and Vishny (1997) illustrate many real problems when dealing with arbitrage strategies and Liu and Longstaff (2002) prove that, under adequate assumptions, margins can make it convenient to abandon available arbitrage strategies in order to optimize the portfolio choice.

Another line of research is related to the arbitrage measurement (see for instance Holden (1995), Chen and Knez (1995), Kempf and Korn (1998), Balbás and Muñoz (1998) or Balbás and López (2000)). In this case authors try to provide us with appropriate measures containing numerical information about the size or degree of arbitrage. If this information incorporates monetary gains associated with the available arbitrage then the effect of some imperfections can be discounted and the theory applies for imperfect markets too. In some sense both approaches are complementary.

The present article is based on the second approach since it uses some measures introduced in Balbás and Muñoz (1998) and Balbás and López (2000) and empirically tests the level of sequential arbitrage in the Spanish bond market.

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The existence of state price densities in a general market, or the existence of a term structure of interest rates (henceforth *TSIR*) in a bond market, is the necessary and sufficient condition to guarantee that the market is arbitrage free, or sequential arbitrage free if dealing with bond markets (see for instance Hansen and Jagannathan (1997) or Ait-Sahalia and Lo (1998) for general markets and Jaschke (1998) for bond markets). This property plays a crucial role when constructing most of the above mentioned arbitrage measures and, specially, those applying in this paper. So, Balbás and López (2000) focus on an arbitrary bond market and introduces several optimization problems minimizing the real price modifications (in percentage) that lead to the existence of a *TSIR* matching the whole set of bond prices. The minimum values are the sequential arbitrage measures. Dual optimization problems give optimal sequential arbitrage strategies and show that these measures may be also interpreted in terms of relative sequential arbitrage earnings. Consequently, when we test the measure values in real bond markets we obtain both optimal relative sequential arbitrage strategies and profits and minimal relative modifications of prices leading to sequential arbitrage free models.

When empirical papers analyze the existence of arbitrage they usually focus on well-known strategies and formulas. For instance, Sternberg (1994) or Kamara and Miller (1995) test the existence of violations of put-call parity, Harris *et al.* (1995) test the price of specific assets or portfolios in different markets, Protopapadakis and Stoll (1983) test the usual relationship between spot and future prices and Lee and Nayar (1993) analyze some relationships between European options and future contracts.

Arbitrage measures also allow us to consider all the available securities and portfolios when testing the market. So, Balbás *et al.* (2000) draw on the Balbás and Muñoz (1998) measure and empirically analyze the level of cross-market arbitrage between the Spanish stock market and its derivative market without previously specifying the exact nature of the arbitrage strategy to be used. Unlike previous studies, these authors globally analyze the market in order to consider all the available arbitrage strategies and choose the optimal one.

Despite the vast empirical literature devoted to analyze the existence of arbitrage in stock and derivative markets, and the vast number of empirical papers focusing on bond prices and the *TSIR*, one misses further empirical evidence concerning the existence of sequential arbitrage in bond markets. Although bond markets and the TSIR deserved and will go on deserving a major attention, see for instance Elton *et al.* (1990), Litterman and Scheinkman (1991), Ilmanen (1992), Bowden (1997), Jaschke (1998), Aniz and Prisman (2000) or Balbás *et al.* (2002) for recent results, there is a clear lack of empirical analyses related to the existence and computation of sequential arbitrage strategies in practice. This fact is probably provoked because there are no well-known arbitrage strategies and formulas when dealing with fixed income securities, unless their derivatives are also involved. In this sense, the global approach of Balbás *et al.* (2000) may be quite suitable when testing bond markets.[4]

The present paper only draws on default free and option free bonds issued by the Spanish government and tests the degree of sequential arbitrage within the time interval 1994-2002. This fact seems to be a clear distinction between this paper and the related empirical literature since there are no risky bonds or derivative securities involved in our analysis. As a consequence, the main conclusions

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4   See also Ioffe (2002).

seem to be robust because they only depend on very general and simple hypotheses and, particularly, no dynamic assumptions are required. We follow the methodology of Balbás et al. (2000) but we use some Balbás and López's measures instead of the Balbás and Muñoz's one. The Balbás and López's measures better capture some properties of sequential arbitrage strategies and bond markets.

The results of our analysis may be useful to traders, managers and researchers. Our test reports practical tools and strategies, the level of sequential arbitrage and it also yields further evidence about market efficiency outside the USA.

We concentrate on the Spanish market *"Mercado de Deuda Anotada"* and the database provided The Bank of Spain, which contains prices, liquidity and other significant information. It is important to point out that Spanish financial markets present appropriate characteristics such as their computerized trading system, their phenomenal recent growth and the positive results that have been obtained when other tests of efficiency have been implemented. For instance, Lee and Mathur (1999) show that the random walk hypothesis cannot be rejected in the Spanish futures markets, while the fulfillment of this hypothesis is not so clear when dealing with another European futures markets.

Despite the comment above the results of our analysis are in the line of those reported by Balbás *et al.* (2000), i.e., the existence of sequential arbitrage cannot be rejected. Hence, global tests based on the theoretical foundations of asset pricing may indicate inefficiency despite previous tests of standard and well-known strategies, simple technical trading rules, or related techniques may have reflected just the opposite.

Besides, it must be also pointed out that the number of sequential arbitrage opportunities and the value of the sequential arbitrage measures significantly increased in 1998, when important innovations were implemented in the market and, amongst other new possibilities, agents began trading each whole bond and its coupons (strips) separately. Consequently, it seems that the inexperience associated with financial innovations may lead to inefficiencies in the market.

Finally, as pointed out in Balbás *et al.* (2000), the arbitrage measurement may also be helpful when analyzing arbitrage-free markets since this measurement may yield new practical strategies to invest and price new securities. Following a similar procedure, our analysis may be also useful on those dates reflecting the absence of sequential arbitrage since it may apply when pricing new bonds or diversifying among available bonds.

The article is organized as follows: Section 2 summarizes those theoretical results that were applied when implementing the empirical test. Section 3 analyzes and describes the market, the data, the empirical results and presents some concrete sequential arbitrage strategies that were available in the market. Section 4 summarizes and concludes the article.

2. PRELIMINARIES, NOTATIONS AND THEORETICAL BACKGROUND

This section is devoted to present a brief synopsis of those theoretical results that were applied when testing the degree of arbitrage in the Spanish bond market. Proofs are omitted since almost all of them may be found in Balbás and López (2000) or another appropriate reference that will be provided.

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Consider n arbitrary bonds B_j, $j=1,2,...,n$, available in the market, and denote by $p=(p_1,p_2,...,p_n)$, $p_j > 0$, $j=1,2,...,n$, the vector whose components are their current prices. Suppose that $T=\{t_1,t_2,...,t_m\}$ represents the set of future dates in which bondholders will receive the corresponding payoff and denote by $a_{ij}>0$ the amount of money paid by B_j, at t_i, $i=1,2,...,m$, $j=1,2,...,n$. In order to ensure that all the statements of this section hold we have to impose the following weak inequality whose economic interpretation is obvious

$$\sum_{i=1}^{m} a_{ij} > p_j$$

for every $j=1,2,...,n$. Consider finally that A represents the $m \times n$ matrix whose rows are $A_i = (a_{i1},a_{i2},...,a_{in})$, $i=1,2,...,m$, and $\widetilde{A} = \begin{pmatrix} A_0 \\ A \end{pmatrix}$ represents the $(m+1) \times n$ matrix obtained by adding A plus a first row equal to $A_0 = -p$. If $X=(x_1,x_2,...,x_n)$ represents the portfolio composed of x_j units of B_j, $j=1,2,...,n$, then pX^T equals the current price of X, AX^T equals its future payoffs and $\widetilde{A}X^T$ equals the whole set of cash flows of X.[5]

The following matrices, whose dimensions are $m \times m$ and $m \times (m+1)$ respectively, will also play an important role in the analysis

$$I_m^* = \begin{pmatrix} 1,0,0,...,0 \\ 1,1,0,...,0 \\ .... \\ 1,1,1,...,1 \end{pmatrix}, \qquad I^{**} = \begin{pmatrix} 1,1,0,...,0 \\ 1,1,1,...,0 \\ .... \\ 1,1,1,...,1 \end{pmatrix}$$

and I_{m+1}^* will be similar to I_m^* but with $m+1$ rows and columns.

We follow the previous literature in order to introduce the concept of sequential arbitrage.

Definiton 1. X is said to be a sequential arbitrage portfolio if $I_{m+1}^* \widetilde{A}X^T \neq 0$ and $I_{m+1}^* \widetilde{A}X^T \geq 0$.[6]

It is known that the sequential arbitrage absence may be characterized by the existence of decreasing and positive discount factors (or a *TSIR*). The statement below clarifies this idea.

Theorem 1. The model is sequential arbitrage free if and only if there exists $\mu=(\mu_1,\mu_2,...\mu_m)$ such that $1>\mu_1> \mu_2>...>\mu_m>0$ and $\mu_A=p$.

In order to measure the level of sequential arbitrage Balbás and López (2000) introduce some extensions of Definition 1. The authors adopt the concept *"arbitrage of the second type"* in the line of Ingersoll (1989), Chapter 2. Mainly, this sort of sequential arbitrage implies a positive income at the current date.

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5   As usual, given an arbitrary matrix M, the transpose of M will be denoted by M^T.
6   As usual, given an arbitrary matrix M, the inequality $M \geq 0$ means that M does not contain any negative element. Similar notations will appear in similar cases. Notice that, according to Definition 1, if a sequential arbitrage portfolio implied any negative cash flow then it should be overcome by the amount of money previously received.

Definition 2. X is said to be a sequential arbitrage portfolio of the second type if $pX^T<0$ and $I^{**}\widetilde{A}X^T \geq 0$. X is said to be a strong sequential arbitrage portfolio if $pX^T<0$ and $I_m^*AX^T \geq 0$.

Obviously, strong sequential arbitrage portfolios are also sequential arbitrage portfolios of the second type, but the converse does not necessarily hold. The absence of strong and second type sequential arbitrage is characterized by the following result (see Balbás and López (2000) for a co0mplete proof).

Theorem 2. The model is second type (respectively, strong) sequential arbitrage free if and only if there exists $\mu=(\mu_1,\mu_2,...\mu_m)^7$ such that $1>\mu_1\geq \mu_2\geq...\geq\mu_m\geq0$ (respectively, $\mu_1\geq \mu_2\geq...\geq\mu_m\geq0$) and $\mu A = p$.

Balbás and López (2000) establish the existence of solution for the minimax problem

$$Min\left[Max\left\{ \frac{p_j - q_j}{p_j}; j = 1,2,...,n\right\}; q = \mu A, q \leq p, \mu_1 \geq \mu_2 \geq \cdots \geq \mu_m \geq 0 \right] \tag{1}$$

$q=(q_1,q_2,...,q_n)$ and $\mu=(\mu_1,\mu_2,...,\mu_m)$ being the decision variables. Denoting by p_{**} and μ_{**} the solution of (1) and by

$$\pounds_{**} = Max\left\{ \frac{p_j - p_{**j}}{p_j}; j = 1,2,...,n\right\}$$

the minimax value, $\pounds_{**}$ may be understood as a measure of the degree of strong sequential arbitrage since, according to Theorem 4, $\pounds_{**}$ is the minimum fall (in percentage) of real prices that leads to a strong sequential arbitrage free model. It may be proved that $0 \leq \pounds_{**} <1$, $\pounds_{**} = 0$ if and only if the model is strong sequential arbitrage free, and $0 < p_{**j} \leq p_j$, $j=1,2,...,n$.

The measure $\pounds_{**}$ may also be interpreted in terms of arbitrage profits. In fact, assume that X is a strong sequential arbitrage portfolio whose short position X has a price equal to one dollar.[8] It may be proved that the price of X^+ verifies $pX^{-T} - pX^{+T} \leq \pounds_{**}$ and, moreover, there exists a strong sequential arbitrage portfolio X_{**} such that $pX^{-T} =1$ and $pX^{-T} - pX^{+T} = \pounds_{**}$. Thus, X_{**} maximizes the profit generated by a strong sequential arbitrage portfolio whose short position has a value equal to one dollar and, moreover, the optimal profit equals $\pounds_{**}$ dollars.

X_{**} also maximizes another interesting objectives that may be interpreted as relative arbitrage earnings. For instance it maximizes the ratio between sequential arbitrage profits and the price of the sold bonds, the ratio between sequential arbitrage profits and the price of the purchased bonds and the ratio between sequential arbitrage profits and the total traded value (i.e.

$$\frac{pX_{**}^{-T} - pX_{**}^{+T}}{pX_{**}^{-T}} \geq \frac{pX^{-T} - pX^{+T}}{pX^{-T}} \ , \ \ \frac{pX_{**}^{-T} - pX_{**}^{+T}}{pX_{**}^{+T}} \geq \frac{pX^{-T} - pX^{+T}}{pX^{+T}} \ \ \text{and} \ \ \frac{pX_{**}^{-T} - pX_{**}^{+T}}{pX_{**}^{-T} + pX_{**}^{+T}} \geq \frac{pX^{-T} - pX^{+T}}{pX^{-T} + pX^{+T}}$$

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7   Balbás and Lopéz (2000) establish the link between μ and several Term Structure of Interest Rates proxies. Thus, they propose new methods to generate the *TSIR* whit the information contained in μ

8   As usual, α⁺=*Sup*{α,0}, α⁺=*Sup*{0,-α} and α=α⁺-α⁻ for every α∈ IR. Similar notations are also used for vectors and matrices.

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hold for every strong sequential arbitrage portfolio X). Furthermore, the first ratio optimal value equals $£_{**}$ (see Balbás and Muñoz (1998) and Balbás and López (2000)).

In order to simplify the notation we will define $h_{**} = X_{**}^-$ and $k_{**} = X_{**}^+$. Hence h_{**} and k_{**} are portfolios without short positions such that k_{**} sequentially dominates h_{**}, the price of k_{**} is $1 - £_{**}$, the price of h_{**} is 1 and k_{**} - h_{**} is the optimal strong sequential arbitrage portfolio. The arbitrage arises because those bonds included in k_{**} are cheap since they verify

$$p_{j**} = p_j \tag{2}$$

while the bonds composing h_{**} are expensive since

$$p_{**j} = p_j(1 - £_{**}) \tag{3}$$

According to the minimax condition, the minimum relative fall that prevents the arbitrage makes the price of those bonds included in h_{**} decrease in the percentage indicated by $£_{**}$. If B_j is neither included in k_{**} nor included in h_{**} then $p_j(1 - £_{**}) \leq p_{**j} \leq p_j$.

Things are more complex when dealing with second type sequential arbitrage portfolios because in this case the minimax condition does not necessarily hold. However, as long as the model is not second type sequential arbitrage free, we can still prove the existence of a second type sequential arbitrage portfolio X_* that maximizes the ratio between the arbitrage income and the value of the short position, the long position or the total traded value. $£_*$ will denote the first ratio optimal value. The portfolio X_* achieves the form $X_* = h^*$ - k^* and h^* being its long and short position respectively. h_* may be normalized so that its price can equal one in which case the price of k_* takes the form $1 - \widetilde{£}_*$, $\widetilde{£}_*$ verifying the constraints

$$0 \leq £_{**} \leq \widetilde{£}_* \leq £_* < 1 \tag{4}$$

There exists a family of prices $p_* = (p_{*1}, p_{*2}, \ldots, p_{*n})$ providing a second type sequential arbitrage free model and such that

$$p_{*j} = p_j \tag{5}$$

if B_j belongs to k_*,

$$p_{*j} = p_j(1 - \widetilde{£}_*) \tag{6}$$

if B_j belongs to k_* and $p_j(1 - \widetilde{£}_*) \leq p_{*j} \leq p_j$ otherwise. Whence, $\widetilde{£}_*$ may also be understood as a measure of the level of second type sequential arbitrage since it provides us with the fall (in percentage) of prices that makes the income given by the optimal second type sequential arbitrage portfolio vanish. Furthermore $\widetilde{£}_*$ is not only a "pricing error" committed by the market since the inequality $\widetilde{£}_* < £_*$ (see (4)) shows that $\widetilde{£}_*$ is a lower bound for the optimal relative second type sequential arbitrage income.

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3. MARKET, DATA, EMPIRICAL RESULTS AND EXAMPLES

The Spanish Treasury issue two types of instruments, *Letras del Tesoro* (Treasury bills) wich are issued at discount with 6-, 12 and 18-moth maturities and Bonos and *Obligaciones del Estado* (Treasury bonds and notes) which bear annual coupon payments and are issued at 3-, 5-, 10-, 15- and 30-year maturities.

Our test focused on the so called *"Mercado de Deuda Anotada"* (*MDA* in what follows), the most liquid market when trading default free and option free bonds issued by the Spanish government. Some bonds with risk premium or embedded option are also available in *MDA* but we did not consider them in order to avoid as much noise as possible. *MDA* is organized by The Bank of Spain and this institution elaborates a complete database containing each type of bond with its corresponding characteristics, daily prices, volume and other significant information. Our test was implemented by employing this database.

Respect daily price, this database records information over maximum, minimum and average traded price for each issue. In our analysis we have used the average traded price to test the existence of sequential arbitrage opportunities.

We have focused on Years 1994, 1998 and 2002 and day by day $\widetilde{£}_*$ and $£_{**}$ have been computed, along with X_{**}, h_{**}, k_{**}, X_*, h_* and k_*. There is not any special reason to choose 1994. We just tried to test an arbitrary year and made a random decision. On the contrary, 1998 reflects several financial innovations that can provoke unusual behaviours of the market. On July 1997, the Spanish Market bond started to issue strippable bonds. For this reason, in 1998 agents have the possibility to trade each whole bond and its strips separately. A general bond is strippable if its coupons (*Small Strips*) and principal (*Large Strip*) may be separately traded.

In order to confirm that 1994 is an arbitrary year we selected a brief sample of days within the time interval 1995-1997 and the results were analogous to those obtained for 1994. Besides, if we eliminate strips from the database of 1998 then we obtain similar results once more. Finally we choose 2002 because in this year the Euro replaced the *Peseta* and in this way we can study the arbitrage measure evolution along a four 4-year period.

Tables 1 and 2 summarize the results. A first remark should point out that many values of $£_{**}$ and $\widetilde{£}_*$ are almost equal. Although from theoretical viewpoints there are some differences between the concepts of strong sequential arbitrage and second type sequential arbitrage, the empirical test seems to reveal that both approaches may lead to analogous results in practice. Tables 1 and 2 show that the levels of both types of arbitrage are closely related.

Tables 1 and 2 also illustrate that the sequential arbitrage existence cannot be globally rejected. Since the test only deals with default free and option free bonds it is clear that the lack of an homogeneous fiscal treatment for all the available bonds or the lack of intradaily perfectly synchronized data are the unique reasons that might justify values attained by $£_{**}$ and $\widetilde{£}_*$ over 20 basic points.

With respect to the second caveat, it was not possible for us to obtain intradaily perfectly synchronized prices. Nevertheless, it is important to recall that $£_{**}$ represents the minimum variation of prices that prevents the strong sequential arbitrage and, according to

Table 1

Year	1994	1998	2002	% over days that $\widetilde{£}_*>0$		
Number of days tasted	247	246	254			
Number of days that $\widetilde{£}_*>0$	42	229	246			
Value of $\widetilde{£}_*$				**1994**	**1998**	**2002**
$\widetilde{£}_*<0.001$	24	76	158	57.14	33.19	64.20
$0.001\leq\widetilde{£}_*<0.0025$	15	65	56	35.71	28.38	22.80
$0.0025\leq\widetilde{£}_*<0.005$	1	67	20	2.38	29.26	8.13
$0.005\leq\widetilde{£}_*<0.01$	1	18	9	2.38	7.86	3.66
$0.01\leq\widetilde{£}_*<0.02$	1	2	2	2.38	0.87	0.81
$0.02\leq\widetilde{£}_*<0.05$	0	1	1	0	0.44	0.41
$0.05\leq\widetilde{£}_*<0.08$	0	0	0	0	0	0

Table 2

Year	1994	1998	2002	% over days that $£_{**}>0$		
Number of days tasted	247	246	254			
Number of days that $£_{**}>0$	41	228	246			
Value of $£_{**}$				**1994**	**1998**	**2002**
$£_{**}<0.001$	24	76	159	58.54	33.33	64.63
$0.001\leq£_{**}<0.0025$	14	65	55	34.15	28.51	22.36
$0.0025\leq£_{**}<0.005$	1	66	20	2.44	28.95	8.13
$0.005\leq£_{**}<0.01$	2	18	9	4.88	7.89	3.66
$0.01\leq£_{**}<0.02$	0	2	2	0	0.88	0.81
$0.02\leq£_{**}<0.05$	0	1	1	0	0.44	0.41
$0.05\leq£_{**}<0.08$	0	0	0	0	0	0

the empirical evidence concerning the volatility of bond prices and interest rates, a few values attained by $£_{**}$ are larger than expected. In this cases, if the lack of synchronization made it difficult to implement sequential arbitrage it would be still obvious that some investors obtained prices and returns significantly different to those provided by a sequential arbitrage free market and significantly better or worse than those generated by any *TSIR*. So, investors who bought bonds included in k_* or k_{**}, (respectively, who sold bonds included in h_* or h_{**}) obtained far better returns than those achieved by investors buying (selling) bonds of h_* or h_{**} (k_* or k_{**}). This assertion is also supported by Expressions (2), (3), (5) and (6) and by the interpretation of the sequential arbitrage measures in terms of relative arbitrage gains.

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Regarding the fiscal treatment, and bearing in mind the interest rate levels in the tested years, even under the most pessimistic scenario it is rather difficult to accept that tax effects may imply

a value of £$_{**}$ greater than 25 basic points in 1994 or 15 basic points in 1998 and 2002.[9] Furthermore, the most favorable fiscal treatment corresponds to strips and the so called *"Letras del Tesoro"* (zero coupon bonds whose maturity is shorter than one year and half) but it will be shown at the end of this section that X_{**} sometimes incorporates *"Letras del Tesoro"* (from now on *LT*) or strips in long position, or coupon bonds in short position.

There were 98 available default and option free bonds in 1994, while these numbers were equal to 137 and 140 in 1998 and 2002 respectively. There are obvious differences between 1994 and 1998. 1998 was characterized by several innovations in *MDA*, strips being the most important (but not the unique) one. We can observe that in 1998 there are more days in which appear sequential arbitrage opportunities. The level of arbitrage in 1998 seems to be bigger, which suggests that the inexperience associated with financial innovations may lead to inefficiencies in the market. If we check 2002 then we can find a clear growth in the level of efficiency.

It is worthwhile to remark that Spanish financial markets have shown a high level of efficiency when several classical tests have been implemented. For example, Lee and Mathur (1999) show that the random walk hypothesis cannot be rejected in the Spanish futures markets, while the fulfilment of this hypothesis is not so clear when dealing with another European future markets. However, our results are in the line of those obtained in Balbás *et al.* (2000), where it is shown that some arbitrage strategies were available in the derivative market whose underlying security consists of the Spanish index *IBEX-35*. As already said, our test and the test of Balbás *et al.* (2000) analyze the market by applying several foundations of asset pricing theory, and without previously specifying the exact nature of the arbitrage strategy to be used. This could be a crucial key to understand these results.

In 1994, in a high percentage (71 per cent) the strong sequential arbitrage opportunities was composed of a short *LT* and a long bond with quite similar maturities. In all this cases the measure is not bigger than 25 basic points, and therefore we could interpret the value of these measures as a fiscal effect.

On the other hand, only a few of available default and option free bonds were actually trade. This circumstance is related with the problem of liquidity. In some cases we could interpret the value of these measures as a liquidity premia.

9 For instance, if we focus on 1998 and 2002, then we can assume an interest rate level whose average value is lower than 5%. Suppose that an agent implements the arbitrage portfolio $X_{**} = k_{**} - h_{**}$. Since the arbitrage profit equals £$_{**}$, capital losses caused by the tax effect should be lower than £$_{**}$. Thus, the net present value of those taxes that must be anticipated when holding k_{**} should be lower than £$_{**}$. According to the Spanish law the percentage of anticipated taxes is not homogeneous and depends on the agent and on her/his portfolio. The most negative situation arises if each coupon c of k_{**} forces the bondholder to anticipate the tax $0.25c$ for one year. According to our empirical results we can assume that k_{**} matures in three years on average and pays an annual coupon $c = 0.05(1 - £_{**})$. Hence, we have that

$$£_{**} \geq 0.05 \times 0.25 \times (1 - £_{**}) \left[\sum_{i=1}^{3} \frac{1}{1.05^i} - \frac{1}{1.05^{i+1}} \right]$$

is a sufficient (but not necessary) condition to guarantee after-tax arbitrage profits. Now, simple manipulations lead to £$_{**} \geq 0.0015$.

The remainder of this section is devoted to present some concrete strong sequential arbitrage portfolios occurred in 1998 and 2002. The purpose of these concrete examples is just to illustrate how sequential arbitrage measures work in practice.

We will present strategies but we will not illustrate that they are really strong sequential arbitrage portfolios because the fulfilment of this property is straightforward. It is sufficient to check that the long position is cheaper than the short position but sequentially dominates. Moreover, it is also easy to check the lack a *TSIR* matching all bond prices unless prices fall in the percentage $£_{**}$ for those bounds composing the short position. All the arbitrage strategies have been normalized so that the price of the short position can equal 10000 Euros.

A first group of examples refers to some strong sequential arbitrage opportunities characterized by surprising properties. In fact, they are extremely simple and generate high values of $£_{**}$. So, Table 3 shows a strategy only composed of two different *LT* whose value at

Table 3
Arbitrage strategies involving LT in 1998

Column 1 indicates the date when the arbitrage occurred Columns 2, 3, 4 and 5 refer to those LT composing the strategy Column 6 indicates $£_{**}$ in basic points					
08-27-98	**LT maturity**	**Value at maturity**	**LT price**	X_{**}	$£_{**}$
	02-19-99	1000	977.24	10.1996	32
	02-26-99	1000	980.43	-10.1996	

Table 4
Arbitrage strategies involving strips in 1998

Column 1 indicates the date when the arbitrage occurred and the strip type Columns 1, 2, 3, 4 and 5 refer to those bonds composing the strategy Column 6 indicates $£_{**}$ in basic points					
03-09-98	**Bond maturity**	**Value at maturity**	**Bond price**	X_{**}	$£_{**}$
Large strip	01-31-13	100000	44392.00	0.2254	61
Small strip	01-31-13	1000	444.75	-22.5474	
04-22-98					
Large strip	01-31-13	100000	45810.00	0.2168	67
Small strip	01-31-13	1000	461.20	-21.6825	
10-22-98					
Large strip	01-31-08	100000	66046.00	-0.1514	45
Small strip	01-31-08	1000	657.46	15.1409	
10-27-98					
Large strip	01-31-29	100000	18265.00	-0.5474	309
Small strip	01-31-29	1000	176.99	54.7495	
12-03-08					
Large strip	01-31-08	100000	68195.00	-0.1466	40
Small strip	01-31-08	1000	679.18	14.6638	
12-30-98					
Large strip	01-31-08	100000	69712.00	-0.1434	60
Small strip	01-31-08	1000	692.89	14.3447	

Table 5
Arbitrage strategies involving strips in 2002

Column 1 indicates the date when the arbitrage occurred and the strip type
Columns 1, 2, 3, 4 and 5 refer to those bonds composing the strategy
Column 6 indicates £$_{**}$ in basic points

22-01-02	Bond maturity	Value at maturity	Bond price	X_{**}	£$_{**}$
Large strip	07-30-14	100000	52298.00	-0.1912	57
Small strip	07-30-14	1000	520.00	19.1211	
01-30-02					
Large strip	01-31-29	100000	23677.00	0.4166	135
Small strip	01-31-29	1000	240.00	-41.6666	
05-27-02					
Large strip	01-31-29	100000	22445.00	-0.4455	52
Small strip	01-31-29	1000	223.28	44.5533	
05-29-02					
Large strip	01-31-08	100000	75654.00	0.1316	43
Small strip	01-31-08	1000	759.81	-13.1611	
06-06-02					
Large strip	01-31-08	100000	74169.00	0.1317	228
Small strip	01-31-08	1000	758.99	-13.1754	
06-14-02					
Large strip	01-31-08	100000	76669.00	0.1291	102
Small strip	01-31-08	1000	774.61	-12.9097	

Table 6
Strategy 1

February 5, 1998

Arbitrage with pure discount bonds ("LT" and strips) and one coupon bond

The coupon is annual and equals 113

£$_{**}$= 49 basic points

Type of bond	Bond maturity	Value at maturity	Bond price	X_{**}
Coupon bond	01-15-02	1000+last coupon	1239.61	8.0277
LT	01-22-99	1000	961.31	-0.9071
Small strip	01-22-00	1000	921.39	-9.0713
Small strip	01-22-01	1000	879.88	-9.0713
Small strip	01-22-02	1000	838.73	-8.9349

maturity is equal to 1000 Euros. It is obvious that the tax effect or the lack of liquidity cannot explain the existence of these strategies because the involved *LT* were traded almost everyday within the last quarter of 1998 and all the *LT* have an analogous fiscal treatment. On the other hand, Table 4 presents several arbitrage strategies involving another type of zero coupon bonds.

Table 7
Strategy 2

March 26, 1998				
Arbitrage with pure discount bonds and one coupon bond.				
All the zero coupon bonds are strips				
The coupon is annual and equals 113				
£$_{**}$ = 39 basic points				
Type of bond	**Bond maturity**	**Value at maturity**	**Bond price**	X_{**}
Coupon bond	01-15-02	1000+last coupon	1255.521	7.9341
Small strip	01-31-99	1000	967.19	-0.8976
Small strip	01-31-00	1000	929.97	-0.8976
Small strip	01-31-01	1000	888.09	-0.8976
Small strip	01-31-02	1000	849.64	-8.8306

Table 8
Strategy 3

May 4, 1998					
The strategy incorporates several coupon and pure discount bonds. All the involved coupons are annual					
£$_{**}$ = 82 basic points					
Type of bond	**Bond maturity**	**Value at maturity**	**Coupon**	**Bond price**	X_{**}
Coupon bond	01-31-06	1000+last coupon	101.50	1340.89	7.3965
Coupon bond	01-31-05	1000+last coupon	50.00	1026.92	-0.7150
Large strip	01-31-03	100000	0	803280.00	-0.0075
Small strip	01-31-99	1000	0	970.17	-0.7150
Small strip	01-31-00	1000	0	931.24	-0.7150
Small strip	01-31-02	1000	0	847.27	-0.7507
Small strip	01-31-03	1000	0	758.55	-0.7507
Small strip	01-31-05	1000	0	716.80	-0.7507
Small strip	01-31-06	1000	0	682.37	-8.1473

Once again, illiquidity or taxes cannot explain these facts. Tables 3 and 4 only provide a small sample of these type of strategies that were available for a large number of days in 1998. Table 5 concentrates on 2002 and shows that the existence of this type of arbitrage significantly decreased.
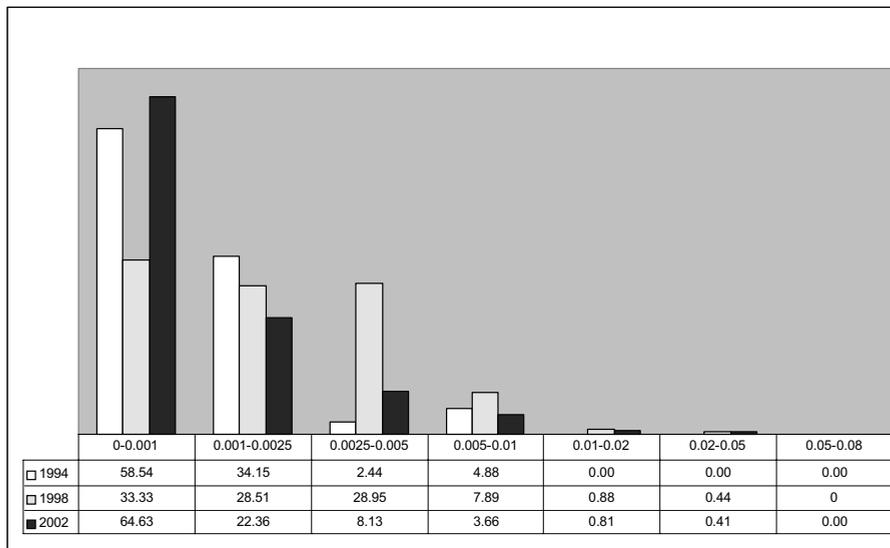
Let us now focus on strong sequential arbitrage strategies involving coupon bonds. The first and second examples (Strategies 1 and 2) incorporate a coupon bond and several pure discount bonds, while Strategies 3 and 4 are more complex and involves several types of bonds in different position (see Tables 6, 7, 8 and 9).

Sometimes, the measure £$_{**}$ may represent liquidity premia for certain issues. In Tables 6 and 7 we can see how the large position is compound for the coupon bond, less liquid that the issues that form the short position. Recent papers study the existence and behaviour of liquidity premia in the Spanish bond market. Alonso *et al.* (2004) estimate

Table 9
Strategy 4

June 2, 2002					
The strategy incorporates several coupon and pure discount bonds.					
All the involved coupons are annual					
$£_{**}$ = 43 basic points					
Type of bond	**Bond maturity**	**Value at maturity**	**Coupon**	**Bond price**	X_{**}
Coupon bond	07-30-02	1000+last coupon	42.5	1037.81	-0.4113
Coupon bond	07-30-04	1000+last coupon	47.5	1044.87	9.5292
Small strip	07-30-03	1000	0	960.00	-0.4288
Small strip	07-30-04	1000	0	920.00	-9.9580

Figure 1
Strong sequential arbitrage mesure evolution along 1994,1998 and 2002

	0-0.001	0.001-0.0025	0.0025-0.005	0.005-0.01	0.01-0.02	0.02-0.05	0.05-0.08
1994	58.54	34.15	2.44	4.88	0.00	0.00	0.00
1998	33.33	28.51	28.95	7.89	0.88	0.44	0
2002	64.63	22.36	8.13	3.66	0.81	0.41	0.00

liquidity premia in the Spanish bond market and found the existence of positive liquidity premia for post-benchmark bond (both strippable and non-strippable). Díaz *et al.* (2006) estimate the structure of liquidity premia in the different maturity sector. It is important to note that only a few of the available default lt and option free bonds were actively traded a during these years, which represents a liquidity problem. For example, strips are more liquids than the associated bond. The existence of liquidity premia could explain the existence of sequential arbitrage opportunities in certain cases. Them we could estimate liquidity premia through these sequential arbitrage measures.

Notice that strategies above have quite different form and some of them are non trivial, which shows the advantages provided by the applied methodology. The main conclusions of this study could not have been reached by testing the presence of concrete sequential arbitrage portfolios.

Overall, we provide a technique for testing the degree of sequential arbitrage in real bond markets and it seems to reveal the existence of available sequential arbitrage portfolios. This assertion is more obvious when markets incorporate significant financial innovations.

4. CONCLUSIONS

This paper has tested the level of sequential arbitrage in the Spanish bond market *MDA* by drawing on the database elaborated by The Bank of Spain. Despite the vast empirical literature devoted to analyze the presence of arbitrage in stock and derivative markets, and the vast number of papers focusing on bond prices and the *TSIR*, it seems to be still scarce the degree of empirical evidence concerning the existence of sequential arbitrage in bond markets. In this sense the present study contributes to complete some possible gaps in the literature.

Empirical papers usually focus on well-known strategies when testing the existence of arbitrage. This technique cannot apply when dealing with bond markets and this is the reason why we followed recent developments that permit us to consider all the available arbitrage portfolios to choose the optimal one. In the present study the optimal strategy is selected by applying some minimax criteria that generate sequential arbitrage measures providing us with the minimum variation of prices leading to sequential arbitrage free models. The sequential arbitrage measures may be also understood as optimal relative sequential arbitrage earnings or incomes.

The first result of our study seems to draw an analogy between the strong sequential arbitrage and the second type sequential arbitrage. Although there are some theoretical differences between both concepts the empirical test seems to reveal that both approaches are quite close in practice.

The empirical results seem to indicate that the presence of sequential arbitrage cannot be rejected. Furthermore, the level of sequential arbitrage significantly raised in 1998, which suggests that the inexperience associated with important financial innovations may provoke inefficiencies in the market. But the level of sequential arbitrage falls as the market has a better knowledge of the behaviour of these innovation, as pointed out by the results obtained where testing Year 2002. These results are surprising by themselves and contrast with those provided by another tests of efficiency applied in the Spanish financial markets. Anyway, it is important to point out that analogous findings were obtained by a previous paper testing the level of arbitrage in the Spanish derivative market by means of global methodologies related to other arbitrage measures.

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Our test only uses default free and option free bonds and, consequently, it is based on very weak hypotheses and the results seem to be quite robust. The lack of synchronization among

prices and a non-homogeneous fiscal treatment for all the available bonds might slightly modify some results but the level of sequential arbitrage seems to be large enough to overcome these imperfections.

Our findings may be useful to traders, managers and researchers because practical procedures to detect the presence of sequential arbitrage have been provided. Moreover, we have computed and analyzed many real sequential arbitrage strategies that were available in the market. Finally, as pointed out by previous papers, the arbitrage measurement may also be interesting when dealing with arbitrage free markets. In such a case the methodology here presented may be extended so that it can apply when pricing new bonds and composing optimal portfolios.

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